

**FastBEM**  **Acoustics**®

**Version 8.0**

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# User Guide

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# Chapter 1. Quick Start

*FastBEM Acoustics*® is a computer program for solving 3-D acoustic wave problems using an advanced boundary element method (BEM) with several fast solution techniques, including the fast multipole method (FMM), adaptive cross approximation (ACA), highly optimized conventional BEM (CBEM), and high-frequency BEM (HFBEM). This chapter introduces the basic concepts and steps in using the program.

## 1.1 Installation

To use the program on the Windows® platform, simply double click the installation program (download from [www.fastbem.com/download.html](http://www.fastbem.com/download.html)) and follow the instructions on the screen. The *installation folder* can look as follows:

**C:\Program Files\FastBEM\FastBEM Acoustics Solver (64)**

where the number “64” appears if the 64-bit program is installed.

To use the program under the Linux OS, unzip and copy the files in the package to a folder where you will launch the program.

## 1.2 Launch the Program

To run the *FastBEM Acoustics*® solver program directly under Windows® 10/11 without using the GUI *FastBEM View*® (see the download page for the package), simply click:

**Start ⇒ All Programs ⇒ FastBEM Acoustics Solver (64) ⇒ FastBEM Acoustics (64)**

or click the program name directly on the Start screen for Windows® 10/11.

You need to provide the name of the path to the project folder where the input files (see below) are located (This can be done easily by copying the folder path in Windows Explorer and pasting it in the DOS window). If the input files are in the current folder (the same folder where the program resides), then simply type “.” (without the quotation marks) and press the Enter key to continue.

You can also copy the solver program *FastBEM\_Acoustics.exe* (or *FastBEM\_Acoustics\_64.exe*) in the installation folder to a folder where you want to launch the program. To run the solver program, double click the program name. Alternatively, you can launch the program by clicking the batch file *FastBEM\_Acoustics.bat* (or *FastBEM\_Acoustics\_64.bat*) which is also available in the installation folder, if the input files are located in the same folder (current folder “.”) as the program file. This batch file can be modified and used to launch the program without the need to type the path of the project folder each time when you launch the program, if the folder path is fixed.

On Linux, use the following command in the folder where the program is located:

`./FastBEM_Acoustics.exe`

to start the program and provide the path to the folder where the input files (see below) are located.

Two basic input files are needed in order to run the *FastBEM Acoustics*® program:

`input.dat` and `input.fmm`

The first one contains the BEM model and the second contains the parameters used by the fast multipole and ACA BEM solvers. These two files are described below (Sample `input.dat` files are available from the installation folder `samples/input_files`).

### 1.3 `input.dat` File

The `input.dat` file contains data for the BEM model to be solved using the *FastBEM Acoustics*® program. This model file can be generated using any third-party FEM/BEM software packages or in-house code by translating the data to the format described below. The input model file has a simple data structure as shown below using the example from the file `input_sample.dat` (in the folder `samples/input_files`).

```

A Sphere Model for Acoustic Scattering Analysis (Line 1)
Complete 1 1 ! Job Type (Complete/Field Only/ATM/Use ATM); Solver Type (1=FMM/2=ACA/3=CBEM/4=HFBEEM); No. Threads (Line 2)
Full 0 0.d0 ! Prob. Space, Symm. Plane, Symm. Plane Property (1=Rigid; -1=Soft) (Line 3)
1200 602 1680 1600 1 ! Nos. of Boundary Elements/Nodes, Nos. of Field Points/Cells, and No. Panels (Line 4)
1 0 ! No. of plane waves (nplane); User defined sources (1=Yes/0=No) (Line 5)
(1., 0.) -1. 0. 0. ! Complex amplitude and direction vector of the plane wave(s) (Line 6)
0 0 ! No. of monopole sources, and no. of dipole sources (Line 7)
343. 1.29 2.d-5 1.d-12 0. ! cspeed, density, ref. pressure, ref. intensity, wavenumber k ratio (Line 8)
54.59 54.59 1 0 0 ! Freq1, Freq2, No. freqs, NOctave, Update BC/freq (1=Yes/0=No) (Line 9)
0 3 1 0 One ! HBIE (1=Yes/0=No), nruleb, nrulef, animation (1=Yes/0=No), Teplot data (All/One) (Line 10)
$ Nodes:
1 0.100000000000000D+01 0.000000000000000D+00 0.000000000000000D+00
2 0.987688340595138D+00 0.156434465040231D+00 0.000000000000000D+00
.....
602 -0.100000000000000D+01 0.000000000000000D+00 0.000000000000000D+00
$ Elements and Boundary Conditions:
1 1 3 2 2 (0.000000000000000D+00, 0.000000000000000D+00) 1
2 1 4 3 2 (0.000000000000000D+00, 0.000000000000000D+00) 1
.....
1200 602 601 596 2 (0.000000000000000D+00, 0.000000000000000D+00) 1
$ Field Points
1 0.100000000D+02 0.000000000D+00 0.000000000D+00
2 0.000000000D+00 0.100000000D+02 0.000000000D+00
.....
1680 0.129994480D+01 -0.102307873D+00 0.000000000D+00
$ Field Cells
1 1 3 81 80
2 3 4 100 81
.....
1600 1680 62 42 1319
$ End of the File

```

**Line 1** in this model file is a comment line, which can be used to provide a short description of the

problem to be solved using the program.

**Line 2** specifies the options for the job, which has two parameters. The first parameter indicates the *Job Type* which has one of the following four options:

*Complete*  
*Field Only*  
*ATM*  
*Use ATM*

*Complete* job type means a complete job starting from solving the BEM model on the boundary to evaluating the results on the field surface (if specified). *Field Only* job type means that the program will only evaluate the results on the field surface(s) using the saved results in the file *output\_result\_saved.dat* (see [Section 1.7](#)). *ATM* job type means that the program will compute the acoustic transfer matrix (ATM) and solve the BEM model using the ATM. *Use ATM* job type means that the program will use the ATM matrix stored in file *output\_ATM\_saved.dat* to solve the BEM model.

The second parameter on line 2 indicates the *Solver Type* and has the value 1, 2, 3 or 4, where

- 1 = fast multipole (FMM) BEM will be used as the solver;
- 2 = adaptive cross approximation (ACA) BEM will be used as the solver;
- 3 = conventional (fast direct) BEM (CBEM) will be used as the solver;
- 4 = high-frequency BEM (HFBEM) will be used.

The third parameter *No. Threads* on line 2 specifies the number of threads of the computer CPU(s) to be used to solve the BEM models. This is for parallel computing on computers with multicore CPUs. If *No. Threads* = 1, one thread will be used and the program will run in sequential mode (no parallel computation). If *No. Threads* is 0, not specified, or specified incorrectly, one half of all the threads of the CPU(s) will be used and the program will run in parallel mode by default.

The remaining portion on line 2 starting at the “?” sign is not read by the program and can be removed or kept for explanation purposes.

Fast multipole BEM solver is the default and preferred solver due to its efficiencies in solving large-scale BEM models. For small to medium sized models (for example, with the number of elements up to 100,000 on a PC with 4 GB of RAM), the ACA BEM solver may converge faster, especially for problems with higher frequencies. The conventional BEM solver is implemented with a fast direct linear equation solver and can be applied for the purpose of comparison or verification, if there is sufficient memory on the computer.

**Line 3** defines the type of space in which a BEM model should be solved by the program. It has one word and two numbers. There are two options for this line, either

*Full*                    0                    0.d0

which instructs the program to solve the BEM model (for an interior or exterior problem) in a 3-D full space; or

*Half*                      *Symm\_Plane*                      *RH\_Coefficient*

which instructs the program to solve the BEM model (for an interior or exterior problem) in a 3-D half space or using a symmetry model, where *Symm\_Plane* is an integer indicating the orientation of the half-space/symmetry plane:

*Symm\_Plane* = 1, 2, or 3 means that the infinite half-space/symmetry plane is  $y$ - $z$ ,  $x$ - $z$  or  $x$ - $y$  coordinate plane, respectively;

and *RH\_Coefficient* is a real constant indicating the property of the half-space/symmetry plane:

*RH\_Coefficient* = 1 means a perfectly *rigid* infinite half-space/symmetry plane;

*RH\_Coefficient* = -1 means a perfectly *soft* infinite half-space/antisymmetry plane.

This coefficient can have any other values between -1 to 1 to represent a surface that has a property between that of a perfectly rigid surface and that of a perfectly soft surface.

**Line 4** has five integers in a *free* format, specifying the numbers of boundary elements, nodes (in this program, it means the corner points defining a constant triangular element), field points inside the domain where the sound pressure need to be calculated, field cells defined by the field points (for plotting the results), and the number of panels (*No. Panels*) in the optional acoustic panel contribution analysis, respectively.

**Line 5** specifies the number of the incident plane waves (*nplane*), and if or not other user defined acoustic sources (other than plane waves and point sources) will be used (An integer 1 = Yes, and 0 = No; See [Section 1.5](#) for more details about the associated file *input.inc*).

In the above sample input file, it indicates that there is one plane incident wave and there are no user defined sources. If *nplane* is not 0, this line is followed by a number of *nplane* lines in the format:

$(Amp\_Real, Amp\_Imag), \quad a_1, \quad a_2, \quad a_3$

to specify the complex amplitude  $\phi_0$  and the three components of the direction vector  $\mathbf{a} = (a_1, a_2, a_3)$  for each plane incident wave.

**Line 6** in the above sample input file specifies an incident wave with amplitude equal to 1 and in the  $-x$  direction. If the direction vector is (0, 0, 0), then a *reverberant condition* is specified.

**Line 7** in the sample input file gives the number of monopole sources (*nsource*) and the number of dipole sources (*ndipole*). In this sample input file, no point sources are specified. If *nsource* is not 0, this line is followed by *nsource* lines in the format:

$(Amp\_Real, Amp\_Imag), \quad x, \quad y, \quad z$

to provide the complex amplitude  $Q_m$  and location  $(x, y, z)$  for each of the monopole sources. If *ndipole* is not 0, the last line is followed by *ndipole* lines in the format:

$(Amp\_Real, Amp\_Imag), \quad x, \quad y, \quad z, \quad d_1, \quad d_2, \quad d_3$

to provide the complex amplitude  $Q_d$ , location  $(x, y, z)$ , and the unit direction vector  $\hat{\mathbf{d}} = (d_1, d_2, d_3)$  for each of the dipole sources.

**Line 8** in the sample input file gives the values of the speed of sound ( $c$ ), mass density ( $\rho$ ) of the

acoustic medium, reference sound pressure  $\phi_{ref}$ , reference sound intensity  $I_{ref}$  (see [Section 1.8](#)), and complex wavenumber ratio ( $k$  ratio) that can be used to introduce a small damping in the acoustic medium (see [Section 5.10](#)). Users need to make sure that the units used for these constants are consistent with those for the BEM model.

**Line 9** specifies the starting and ending frequencies of the frequency range, number of frequencies, if or not the 1/Nth-octave band will be used (For example, if  $N = 0$ , then no octave band will be used. If  $N = 3$ , then 1/3-octave band will be used; and so on), and if or not the boundary conditions will be updated at each frequencies (An integer 1 = Yes, and 0 = No; See [Section 1.6](#) for more details about the associated file *input.jbc*). For a single frequency analysis, simply set  $Freq1 = Freq2$ , and  $No. Freq = 1$ .

**Line 10** tells the program if or not the dual boundary integral equation (BIE) formulation should be used (1 = Yes and 0 = No. See [Chapter 3](#) for the BIE formulations for acoustic wave problems), the quadrature rules to be used for integration (see [Section 5.3](#) for more information) on the boundary elements for boundary points with the following options:

$$nruleb = 1, 2, 3, 4, 5, 6;$$

and those for integration on the boundary elements for field points with the following options:

$$nrulef = 1, 2, 3, 4, 5, 6;$$

and if or not animation should be generated (*animation* = 1 means Yes and *animation* = 0 means No). The last parameter *Tecplot Data* is related to if or not the Tecplot contour plot data should be saved for all the frequencies (*Tecplot Data* = All) or just for the last frequency (*Tecplot Data* = One, which is also the default). Saving all the Tecplot data for all frequencies and selecting animation can take up large disk space and should be used only when needed and after the results have been verified.

The rest of the records in the model file lists the coordinates of the nodes and element connectivity for the boundary, and those for the field points and field cells, as indicated by the preceding comment line. For the boundary nodes, the format of the record is (in free format):

$$Node\ Number, \quad x, \quad y, \quad z$$

For the boundary element connectivity, the format is:

$$Element\ Number, \quad Node\ 1, \quad Node\ 2, \quad Node\ 3, \quad BC\_Type, \quad BC\_Value, \quad Panel\ Number$$

where *Node 1*, *Node 2* and *Node 3* are the global numbers of the three corner nodes defining the element, and

$BC\_Type = 1$  means the pressure is given,

$BC\_Type = 2$  means the normal particle velocity is given,

$BC\_Type = 3$  means the specific impedance is given.

*BC\_Value* is a complex number in the form (*Real part*, *Imaginary part*) that stores the given value of the pressure, normal velocity or specific impedance at the center of that element.

*Panel Number* is optional for acoustic panel contribution analysis. If specified, it indicates the number of the panel to which the element belongs.

For the boundary element connectivity, the nodes should be arranged in such a way that the *normal of each element points away from the acoustic medium* (for example, into the structure) following the right-hand rule (see Figure 3.3).

The data for the field points and field cells are defined next, where the sound pressure and normal component of the particle velocity will be evaluated after the solution of the BEM system of equations is obtained. The field cells are applied only for the purpose of plotting the results on the field surface and can be omitted if no plots are needed.

Note that 3-node triangular surface elements (T3) are used on the boundary of the structure, while 4-node quadrilateral elements (Q4) are used in defining the field cells. As a special case, a field cell can also have a 3-node triangular shape if the 3rd and 4th nodes share the same field point.

### 1.4 Input.fmm File

The file *input.fmm* contains parameters used for the fast multipole and ACA BEM and the iterative linear equation solver. The iterative equation solver is used in both the fast multipole BEM and the ACA BEM. In general, the default values of these parameters can be used and need not be changed each time when solving a new BEM model.

The default values and meanings of the parameters specified in file *input.fmm* are given in the file directly, which is shown below:

```

Parameters for the fast multipole method:
100    10    6    1.E-4    400000000    500    !maxl    levmx    nexp    tolerance    nstk    iteration
60000  50000  100    0    10    !ncellmx    nleafmx    maxk    ifmm    nexpf

-----

Definitions of the above parameters:

maxl:    maximum number of elements in a leaf
levmx:   maximum number of tree levels
nexp:    order of fast multipole expansions (p)
tolerance: tolerance for convergence used in the iterative solver
nstk:    size of the array used to store coefficients in preconditioner (value > 0 or = 0)
iteration: maximum number of iterations allowed for the iterative solver
ncellmx: maximum number of cells allowed in the tree
nleafmx: maximum number of leaves allowed in the tree
maxk:    maximum dimension of Krylov subspace used in the iterative solver (This was named mxl in earlier versions)
ifmm:    apply FMM in field evaluation (0-no (direct method is used); 1-yes)
nexpf:   number of expansion terms in the field evaluation if FMM is used

-----

Default values of these parameters:

100    10    6    1.E-4    400000000    500    !maxl    levmx    nexp    tolerance    nstk    iteration
60000  50000  100    0    10    !ncellmx    nleafmx    maxk    ifmm    nexpf
    
```

Detailed descriptions of some of the parameters are provided in [Chapter 5](#), in which their effects on the accuracy and efficiency of the BEM solutions are also discussed.

The program will stop if one or several of the above parameters (e.g., *ncellmx* and *nleafmx*) are not adequate and messages suggesting changes will appear on the screen. Simply adjusting (e.g., increasing) the values of these parameters can often fix the problem.

## 1.5 *Input.inc* File

If you want to specify a user-defined acoustic source (such as line-, area-, volume-distributed sources) that cannot be described by either plane incident waves or point monopole and dipole sources, you need first change the value of the second integer in [Line 5](#) of the *input.dat* file to 1 and then provide the user-defined source in a new input file named *input.inc*. Note that both the number of plane waves and that of the point sources should be specified as 0's if they are not present. The format of this *input.inc* file is shown in the following example (See the sample *input.inc* file in the installation folder):

```

Incident Field at Frequency No.:      1
Boundary element, phi_inc, q_inc:
1  (0.547192841743995D+00,-0.837006567443847D+00) (0.833569353920684D+00, 0.544945764231611D+00)
2  (0.547192841743995D+00,-0.837006567443847D+00) (0.833569353920684D+00, 0.544945764231611D+00)
3  (0.547192841743995D+00,-0.837006567443847D+00) (0.833569353920684D+00, 0.544945764231611D+00)
4  (0.547192841743995D+00,-0.837006567443847D+00) (0.833569353920684D+00, 0.544945764231611D+00)
.....
Incident Field at Frequency No.:      2
Boundary element, phi_inc, q_inc:
1  (-0.880861007794179D+00, 0.473374994003510D+00) (-0.471431073310387D+01,-0.877243740379322D+01)
2  (-0.880861007794179D+00, 0.473374994003510D+00) (-0.471431073310387D+01,-0.877243740379322D+01)
3  (-0.880861007794179D+00, 0.473374994003510D+00) (-0.471431073310387D+01,-0.877243740379322D+01)
4  (-0.880861007794179D+00, 0.473374994003510D+00) (-0.471431073310387D+01,-0.877243740379322D+01)
.....

```

In this file, following two comment lines, the element number, values of the sound pressure and its normal derivative for the source on the element are given for all boundary elements. The records are repeated for each frequency value as specified in the *input.dat* file.

## 1.6 *Input.jbc* File

If the boundary conditions need to be updated at each frequency in a frequency response analysis, such as the case of computing the sound fields due to the harmonic responses of a structure, you need first change the value of the last integer in the line specifying the frequencies in the *input.dat* file to 1 and then provide the BC values in a new input file named *input.jbc*. The format of this *input.jbc* file is shown in the following example (See the sample *input.jbc* file in the installation folder):

```

$ BC Values for Frequency No. 1:
  54.59      !Frequency (Hz)
$ Boundary Elem No. BC Type BC Values
  1          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  2          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  3          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  4          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  .....
$ BC Values for Frequency No. 2:
  545.9      !Frequency (Hz)
$ Boundary Elem No. BC Type BC Values
  1          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  2          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  3          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  4          2          (-0.100000000000000D+01, 0.000000000000000D+00)
  .....

```

In this file, following one comment line, the frequency value is given; following another comment line, the element number, BC type and BC value on the element are given for all boundary elements. The records are repeated for each frequency. If the values of the frequencies given in this file are different from the ones in the *input.dat* file or calculated by the program, then the values in this *input.jbc* file will override the other values of the frequencies. This is useful when the frequency values are neither uniformly spaced nor controlled by the 1/Nth-octave band. Therefore, they cannot be specified by the parameters in the *input.dat* file. However, the total number of frequencies is still governed by the number of frequencies given in the *input.dat* file.

## 1.7 File Management and Output Files

**FastBEM Acoustics**® program uses folders to manage different projects or BEM models. The folder name is used as the project name. Within each folder, all the input and output files have the same names as in other folders for other projects. So users should get familiar with this approach and use different folders to store the input and output files for different projects or BEM models.

Several output files are generated when the program executes successfully. The definitions and usages of these files are described in the following:

*output.log*: This file contains a copy of the output to the screen when the program executes. It can be used to check the parameters used in solving the BEM model.

*output\_freq\_responses.plt*: This file contains values of the maximum amplitude of the sound pressure and sound pressure level (SPL) on the field surface, the radiated sound power and sound power level (PWL) of the model at each frequency, which can be used to plot frequency response curves using any curve plotting software.

*output\_result.dat*: This file has complete lists of the solutions on all the boundary elements and at all the field points, including sound pressure, SPL, particle velocity (normal component) and sound intensity level (SIL), as well as the sound power and PWL, at each frequency.

One can use the data in this file to process the BEM results using own in-house code, or other software such as Matlab, and so on.

*output\_tecplot.plt:*

This file contains the results in the format of the software Tecplot®, that can be used to view the BEM model and results in Tecplot®. The results include the real part, imaginary part and amplitude of the sound pressure and normal component of the particle velocity, SPL and SIL on the boundary of the structure and at the field points. By default, *these results are for the last frequency only, if multiple frequency analysis is conducted*, due to the potentially large size of this file. To save the Tecplot® data for results at all the frequencies, set the parameter *Tecplot Data = All* in the *input.dat* file (see Section 1.3).

*output\_ATM\_Saved.dat*

This file stores the acoustic transfer matrix (see Chapter 2 for the exact definition) when the job type option *ATM* is used. The data in the file are in ASCII text format and can be opened and read using any word processor to see the format used in the file for storing the coefficients of the matrix. Note: Starting with Version 6, file *output\_ATM.dat* and file *output\_ATM\_Saved.dat* are combined. The former is no longer needed.

*output\_freq\_responses\_panels.plt*

When the acoustic panel contribution analysis is requested (*No. Panels > 1*) and panels are defined, this files stores the computed sound pressure and the acoustic *contribution factor* from each panel and at each frequency for the field point where the pressure magnitude is maximum.

To re-run the program with the job type option *Field Only*, one need to use a file named:

*output\_result\_saved.dat:*

This file is a copy of the *output\_result.dat* file containing the results on the boundary of a previously solved BEM model. The BEM model in this file should match with the model in the *input.dat* file to be used to re-run the job. Only field points and cells can be different in this new *input.dat* file from the initial one.

To re-run the program with the job type option *Use ATM*, one need to use the file *output\_ATM\_saved.dat*. The BEM model in this file should match with the model in the *input.dat* file to be used to re-run the job. The field points and field cells should also be the same as in the original model. Only the velocity/pressure BC values or direction of the incident waves can be different in this new *input.dat* file from the initial one.

**Animations** of the BEM results in the time domain at a given *single frequency* can be generated using results in file *output\_tecplot.plt*. To do this, set *Animation = 1* in the *input.dat* file. After the solution is done, use the **Tools/Animate** command in Tecplot® and select results in zones from *Animation Frame 1* to *Animation Frame 21* corresponding to a frequency. The animations are for one full period (one full wavelength) using 21 frames of the BEM results that have been converted to the time

domain (with the time factor  $e^{-i\omega t}$ , where  $i = \sqrt{-1}$ ). Again, animation results can take up large amount of disk space and should be invoked only when needed and after the results are verified.

## 1.8 Definitions of the Terms

Definitions of the acoustic related quantities computed by the *FastBEM Acoustics*® program and listed in the output files are given in the following.

The *sound pressure level* (SPL) is defined by:

$$\text{SPL} = 20 \log_{10}(\phi / \phi_{ref}), \quad (1.1)$$

with unit dB, where  $\phi$  is the sound pressure, and  $\phi_{ref}$  the reference sound pressure level which, for example, has a value of  $\phi_{ref} = 20 \times 10^{-6} \text{ N/m}^2 = 20 \mu\text{Pa}$  if the acoustic medium is air and a value of  $\phi_{ref} = 1 \times 10^{-6} \text{ N/m}^2 = 1 \mu\text{Pa}$  if the medium is water.

The *sound power* radiated from a structure is defined by:

$$W = \frac{1}{2} \text{Re} \left\{ \int_S (\phi \bar{v}_n) dS \right\}, \quad (1.2)$$

with unit watts, where  $S$  is the boundary of the structure,  $\bar{v}_n$  the complex conjugate of the normal component of the velocity, and  $\text{Re} \{ \}$  indicates the real part of the result.

The *sound power level* (PWL) is defined by:

$$\text{PWL} = 10 \log_{10}(W / W_{ref}), \quad (1.3)$$

with unit dBW, where the reference sound power  $W_{ref} = 10^{-12}$  watts .

The *sound intensity level* (SIL) is defined by:

$$\text{SIL} = 10 \log_{10}(I / I_{ref}), \quad (1.4)$$

with unit dB, where  $I$  is the *sound intensity* or rate of energy flow across a unit area (or sound power per unit area), and the reference sound intensity  $I_{ref} = 10^{-12}$  watts/m<sup>2</sup> .

# Chapter 2. Overview of the BEM Formulations

The boundary element method [1-5] for solving acoustic wave problems is reviewed and the fast solution methods for solving the BEM systems of equations are introduced in this chapter. Readers interested in the theoretical background of these methods, especially the approaches used in developing this program, should consult the books and papers listed in the section [References](#).

## 2.1 Governing Equation

Consider the acoustic domain  $E$  with boundary  $S$  (e.g., the surface of a structure  $V$ ). Domain  $E$  can be either an interior domain inside  $S$  or an exterior domain outside  $S$  (The latter is shown in [Figure 3.1](#)).

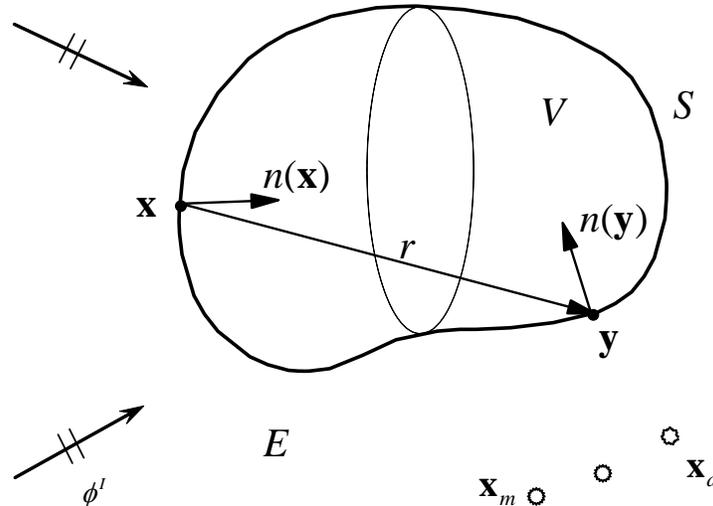


Figure 2.1. An acoustic domain  $E$  outside a structure  $V$ .

We assume the sound pressure in the time domain is in the form  $p(\mathbf{x}, t) = \phi(\mathbf{x}, \omega)e^{-i\omega t}$  (with  $i = \sqrt{-1}$ ) for time harmonic waves, where  $\phi(\mathbf{x}, \omega)$  is the complex sound pressure in the frequency domain and is governed by the following *Helmholtz equation*:

$$\nabla^2 \phi + k^2 \phi + Q_m \delta(\mathbf{x}, \mathbf{x}_m) + Q_d \hat{\mathbf{d}} \cdot \nabla_{\mathbf{x}_d} \delta(\mathbf{x}, \mathbf{x}_d) = 0, \quad \forall \mathbf{x} \in E. \quad (2.1)$$

In the above equation,  $k = \omega / c$  is the *wavenumber* (which can also be a complex number, see [Section 4.10](#)),  $\omega$  the *circular frequency*, and  $c$  the *speed of sound* in the medium. The term  $Q_m \delta(\mathbf{x}, \mathbf{x}_m)$  represents a typical *monopole point source* located at  $\mathbf{x}_m$  (inside the fluid domain  $E$ , [Figure 2.1](#)) with

$Q_m$  being the complex amplitude of the monopole source, and the term  $Q_d \hat{\mathbf{d}} \cdot \nabla_{\mathbf{x}_d} \delta(\mathbf{x}, \mathbf{x}_d)$  represents a typical *dipole point source* located at  $\mathbf{x}_d$  (Figure 2.1) with  $Q_d$  being the complex amplitude and  $\hat{\mathbf{d}}$  being the unit direction vector of the dipole source.

The *acoustic boundary conditions* (BCs) can be classified as follows:

*Pressure is given:*

$$\phi = \bar{\phi}, \quad \forall \mathbf{x} \in S; \quad (2.2)$$

*Velocity is given:*

$$v_n = \frac{1}{i\omega\rho} \frac{\partial \phi}{\partial n} = \bar{v}_n, \quad \forall \mathbf{x} \in S; \quad (2.3)$$

*Specific impedance is given:*

$$Z = \frac{\phi}{v_n} = \bar{Z}, \quad \forall \mathbf{x} \in S; \quad (2.4)$$

where  $\rho$  is the *mass density*,  $v_n$  the normal particle velocity with  $n$  being the outward normal (Figure 2.1),  $Z$  the *specific acoustic impedance* which is a complex number and defined with the particle velocity  $v_n$  going into the surrounding material (in the direction of normal  $n$ ) [6]. The overbars in the above three equations indicate given values. Mixed BCs can also be specified on the boundary of the acoustic domain.

In addition to acoustic sources inside the domain, sources due to *plane incident waves* may also be present. A plane incident wave  $\phi^I$  from infinity (see Figure 2.1) can be defined by:

$$\phi^I(\mathbf{x}) = \phi_0 e^{i\mathbf{x} \cdot \hat{\mathbf{a}}}, \quad (2.5)$$

where  $\phi_0$  is the complex amplitude of the plane incident wave, and the unit vector  $\hat{\mathbf{a}}$  indicates the direction of the incident wave.

If  $\hat{\mathbf{a}} = (0, 0, 0)$  for the incident wave, then

$$\phi^I(\mathbf{x}) = \phi_0, \quad (2.6)$$

which represents a *reverberant condition*.

For the boundary-value problem, we will need to solve Eq. (2.1) at a given frequency  $f$  or wavenumber  $k$ , and under the boundary conditions in (2.2) to (2.4), considering possible incident waves and point sources.

**FastBEM Acoustics®** program can solve both radiation and scattering acoustic wave problems. For a radiation problem, a structure is in vibration and causes disturbances in the acoustic field outside or inside the structure. In this case, the normal component of the particle velocity on boundary  $S$  is specified. For a scattering problem, the structure is assumed to be motionless and an

incoming disturbance (e.g., an plane incident wave or a point source) interacts with the structure and waves are scattered by the structure. The total wave is the sum of the scattered wave and the incident wave or point source for a scattering problem.

**FastBEM Acoustics**® program allows any combination of the boundary conditions with the acoustic sources. For example, one can model a radiation problem with velocity and/or impedance BCs, and in the same time with several plane incident waves and/or point sources in the domain as additional acoustic sources.

For exterior (infinite domain) acoustic wave problems, in addition to the boundary conditions on  $S$ , the field at infinity must satisfy the following *Sommerfeld radiation condition*:

$$\lim_{R \rightarrow \infty} \left[ R \left| \frac{\partial \phi}{\partial R} - ik\phi \right| \right] = 0, \tag{2.7}$$

where  $R$  is the radius of a large sphere covering the structure and  $\phi$  is the radiated wave in a radiation problem or the scattered wave in a scattering problem. Basically, the Sommerfeld radiation condition says that any acoustic disturbances caused by the structure (either radiated or scattered) should die out at infinity based on the energy consideration.

The BEM formulations satisfy the Sommerfeld radiation condition *automatically* at the infinity. Therefore, no discretization at any other regions, except for the boundary of the structure, is needed in the BEM solution for infinite domain problems. This is a huge advantage of the BEM for solving acoustic wave problems in infinite domains over other domain-based methods.

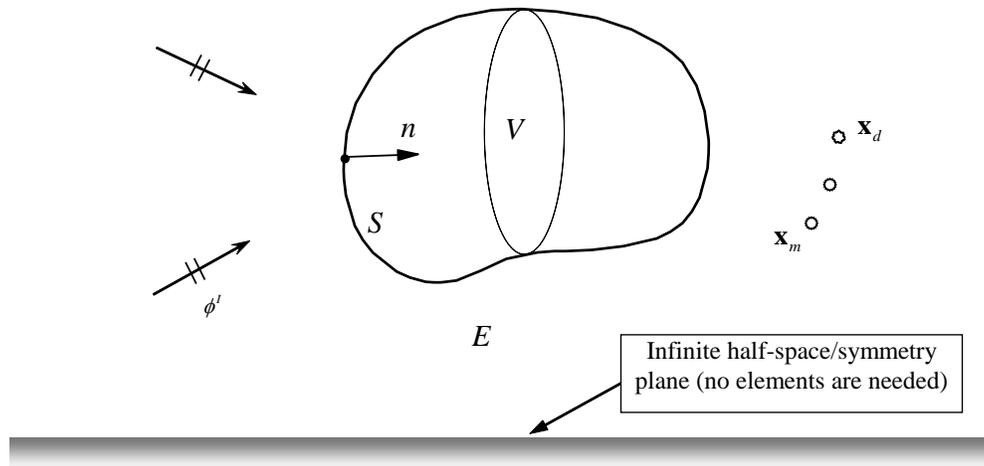


Figure 2.2. An acoustic domain  $E$  in a 3-D half space.

**FastBEM Acoustics**® program can also model half-space acoustic wave problems or problems with a symmetry/antisymmetry plane (Figure 2.2). No boundary elements are needed on the symmetry or infinite half-space plane, because of the use of the special half-space Green’s functions. Perfectly rigid, soft or any other infinite half-space/symmetry plane can be modeled (See Section 1.3 for how to specify the related parameters in *input.dat* file).

## 2.2 Boundary Integral Equation Formulations

The Helmholtz equation can be transformed into boundary integral equations (BIEs) and then solved using the BEM. The conventional boundary integral equation (CBIE) is given by [5]:

$$c(\mathbf{x})\phi(\mathbf{x}) = \int_S [G(\mathbf{x}, \mathbf{y}, \omega)q(\mathbf{y}) - F(\mathbf{x}, \mathbf{y}, \omega)\phi(\mathbf{y})] dS(\mathbf{y}) + \phi^s(\mathbf{x}), \quad \forall \mathbf{x} \in S, \quad (2.8)$$

in which the constant  $c(\mathbf{x}) = 1/2$ , if  $S$  is smooth around point  $\mathbf{x}$ ,  $q = \partial\phi / \partial n = i\omega\rho v_n$ , and the kernels (Green's functions) are given by:

$$G(\mathbf{x}, \mathbf{y}, \omega) = \frac{1}{4\pi r} e^{ikr}, \quad F(\mathbf{x}, \mathbf{y}, \omega) \equiv \frac{\partial G(\mathbf{x}, \mathbf{y}, \omega)}{\partial n(\mathbf{y})}, \quad (2.9)$$

for 3-D full-space problems, with  $i = \sqrt{-1}$ ,  $r$  is the distance between the source point  $\mathbf{x}$  and field point  $\mathbf{y}$  (Figure 2.1). In Eq. (2.8), the term

$$\phi^s(\mathbf{x}) \equiv \phi^l(\mathbf{x}) + Q_m G(\mathbf{x}, \mathbf{x}_m, \omega) + Q_d \hat{\mathbf{d}} \cdot \nabla_{\mathbf{x}} G(\mathbf{x}, \mathbf{x}_d, \omega) \quad (2.10)$$

includes all the source terms, including a typical plane incident wave  $\phi^l$ , monopole  $Q_m$  and dipole  $Q_d$  terms. CBIE (2.8) can be employed to solve for the unknown  $\phi$  and  $q$  on  $S$ . It is well known that this CBIE has a major defect for exterior domain problems, that is, it has nonunique solutions at a set of *fictitious eigenfrequencies* or *irregular frequencies* associated with the resonate frequencies of the corresponding interior problems [7]. This difficulty is referred to as the *fictitious eigenfrequency difficulty* [5, 8-11]. A remedy to this problem is to use the normal derivative (hypersingular) BIE in conjunction with this CBIE.

The hypersingular boundary integral equation (HBIE) is given by (see Refs. [5, 8-11]):

$$\tilde{c}(\mathbf{x})q(\mathbf{x}) = \int_S [K(\mathbf{x}, \mathbf{y}, \omega)q(\mathbf{y}) - H(\mathbf{x}, \mathbf{y}, \omega)\phi(\mathbf{y})] dS(\mathbf{y}) + q^s(\mathbf{x}), \quad \forall \mathbf{x} \in S, \quad (2.11)$$

where  $\tilde{c}(\mathbf{x}) = 1/2$  if  $S$  is smooth around  $\mathbf{x}$  and  $q^s(\mathbf{x}) = \partial\phi^s(\mathbf{x}) / \partial n(\mathbf{x})$  is the source term. The two new kernels are given by:

$$K(\mathbf{x}, \mathbf{y}, \omega) \equiv \frac{\partial G(\mathbf{x}, \mathbf{y}, \omega)}{\partial n(\mathbf{x})}, \quad H(\mathbf{x}, \mathbf{y}, \omega) \equiv \frac{\partial F(\mathbf{x}, \mathbf{y}, \omega)}{\partial n(\mathbf{x})}. \quad (2.12)$$

For exterior acoustic wave problems, a dual BIE (CHBIE, or composite BIE [8-11]) formulation using a linear combination of the CBIE (2.8) and HBIE (2.11) can be written as:

$$\text{CBIE} + \beta \text{HBIE} = 0, \quad (2.13)$$

where  $\beta$  is a coupling constant. This dual BIE formulation is also called the Burton-Miller BIE formulation [7] for acoustic wave problems and has been shown by Burton and Miller to yield unique solutions for exterior problems at all frequencies, if  $\beta$  is a *complex number*.

Once the acoustic pressure  $\phi$  and  $q = \partial\phi / \partial n$  on  $S$  are obtained, the values of  $\phi$  at any point  $\mathbf{x}$  inside the acoustic domain  $E$  can be determined by applying the following representation integral:

$$\phi(\mathbf{x}) = \int_S [G(\mathbf{x}, \mathbf{y}, \omega)q(\mathbf{y}) - F(\mathbf{x}, \mathbf{y}, \omega)\phi(\mathbf{y})] dS(\mathbf{y}) + \phi^s(\mathbf{x}), \quad \forall \mathbf{x} \in E, \quad (2.14)$$

which can be regarded as a post-processing step (*field evaluation*) in the BEM solution.

## 2.3 Boundary Element Discretization

The discretized equations of CBIE (2.8), HBIE (2.11), or the Burton-Miller's BIE formulation (2.13), can be obtained by discretizing the boundary  $S$  using constant elements (Figure 2.3; Ref. [12]) or other higher-order elements [8-11]. In the *FastBEM Acoustics*® code, constant *triangular* surface elements are used in the discretizations of the boundaries (Figure 2.3). With constant elements, all the singular and hypersingular integrals can be evaluated analytically, which is more efficient and accurate than computing these singular integrals with any numerical quadrature.

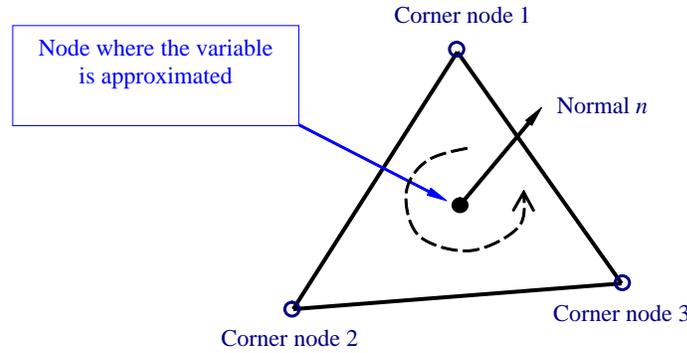


Figure 2.3. A constant triangular boundary element.

The discretized BIEs can be written as:

$$\mathbf{F}\phi = \mathbf{G}\mathbf{v} + \phi^s \quad (2.15)$$

where  $\mathbf{F}$ ,  $\mathbf{G}$ ,  $\phi$ ,  $\mathbf{v}$  and  $\phi^s$  are the two coefficient matrices computed using related kernels, the sound pressure vector and normal velocity vector from each element, and the vector from the source terms, respectively. Applying the boundary conditions and switching the columns, the BEM equation (2.15) can be written as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{Bmatrix}, \quad \text{or} \quad \mathbf{A}\boldsymbol{\lambda} = \mathbf{b}, \quad (2.16)$$

where  $\mathbf{A}$  is the system matrix,  $\boldsymbol{\lambda}$  the vector of unknown boundary variables at the nodes,  $\mathbf{b}$  the known vector, and  $N$  the number of unknowns on the boundary (= number of elements in the case of using constant elements). This system of equations is in complex numbers, that is, all the coefficients and variables are complex numbers and thus the memory requirement is four times as large as its counterpart as in the static problems.

Obviously, the construction of matrix  $\mathbf{A}$  in Eq. (2.16) requires  $O(N^2)$  operations and the size of the required memory for storing  $\mathbf{A}$  is also  $O(N^2)$  since  $\mathbf{A}$  is in general a non-symmetric and dense matrix for the BEM. The solution of system in Eq. (2.16) using direct solvers such as Gauss elimination is even worse, requiring  $O(N^3)$  operations. That is why the conventional BEM for solving the BIEs is slow and inefficient for large-scale BEM problems, despite its robustness in the modeling or meshing stage as compared with other domain-based methods.

## 2.4 Acoustic Transfer Matrix (ATM)

For an acoustic wave problem in which the geometry of the radiating or scattering structure is fixed, the *acoustic transfer matrix* concept can be applied to quickly recalculate the sound field outside the structure when different values of the BC (for a radiation problem) or different directions of the incident wave (for a scattering problem) are applied. In this case, the BEM matrix can be computed and stored first, and then is retrieved for fast field evaluation in another run with different values of the BC, or different directions of the incident wave.

The formulation of the ATM is summarized as follows. Suppose we have  $N_f$  field points. The discretized equations for the sound pressure at these field points can be written as:

$$\phi_f = \mathbf{G}_f \mathbf{v} - \mathbf{F}_f \phi + \phi^s, \quad (2.17)$$

based on Eq. (2.14), where  $\phi_f$ ,  $\mathbf{G}_f$ ,  $\mathbf{F}_f$ ,  $\mathbf{v}$ ,  $\phi$  and  $\phi^s$  are the pressure at the field points, two coefficient matrices, the boundary velocity vector, boundary pressure vector, and the source term vector (including plane incident waves and point sources, see Eq. (2.10)), respectively.

For *radiation* problems (no source terms), the *acoustic transfer matrix* or *vector* can be applied to rapidly optimize the acoustic response at a few field points from a radiating structure, for which the boundary geometry is fixed, but the velocity or pressure profile on the boundary can be modified in order to optimize the acoustic response at the selected field points and given frequencies.

If values of the *boundary velocity* are specified, from Eq. (2.15), the boundary pressure vector is found to be:

$$\phi = \mathbf{F}^{-1} \mathbf{G} \mathbf{v}. \quad (2.18)$$

Substituting this into Eq. (2.17), we have the expression for pressure at the selected field points:

$$\phi_f = [\mathbf{G}_f - \mathbf{F}_f \mathbf{F}^{-1} \mathbf{G}] \mathbf{v} \equiv \mathbf{T} \mathbf{v}, \quad (2.19)$$

where  $\mathbf{T} = \mathbf{G}_f - \mathbf{F}_f \mathbf{F}^{-1} \mathbf{G}$  is defined as the acoustic transfer matrix in the case where the normal velocity on the boundary is given.

Similarly, if values of the *boundary pressure* are specified, the acoustic transfer matrix is given by the following expression:

$$\phi_f = [\mathbf{G}_f \mathbf{G}^{-1} \mathbf{F} - \mathbf{F}_f] \phi \equiv \mathbf{T} \phi, \quad (2.20)$$

where  $\mathbf{T} = \mathbf{G}_f \mathbf{G}^{-1} \mathbf{F} - \mathbf{F}_f$  is defined as the acoustic transfer matrix in the case where the pressure on the boundary is specified.

The acoustic transfer matrix  $\mathbf{T}$  has the dimension of  $N_f \times N$ . Matrix  $\mathbf{T}$  can be computed once and stored for a fixed boundary geometry, fixed locations of the field points and a given frequency, if the job type *ATM* is selected in *FastBEM Acoustics*®.

With a different velocity or pressure distribution on the boundary, the sound pressure at the same field points can be computed readily using expression (2.19) or (2.20), if the job type *Use ATM* is later selected, without having to solve the BEM model again. When  $N_f = 1$  (one field point),  $\mathbf{T}$  is called an *acoustic transfer vector*.

Note that the acoustic sensitivity of the sound pressure at the field points with respect to the velocity  $v_j$  on boundary element  $j$  ( $j = 1, 2, \dots, N$ ) is given by:

$$\frac{\partial \phi_f}{\partial v_j} = \mathbf{T} \frac{\partial \mathbf{v}}{\partial v_j} = [\mathbf{T}]_j. \quad (2.21)$$

That is, the values of the *acoustic sensitivity* to  $v_j$  are given by the  $j$ -th column of matrix  $\mathbf{T}$ . Applications of the ATM and acoustic sensitivity in reducing the noise from a vibrating structure can be found in Ref. [13]. The same conclusion can be drawn for the pressure given case.

For *scattering* problems, the acoustic transfer matrix can be constructed similarly, and computed and stored for future use with different acoustic sources. The expressions are

$$\phi_f = [-\mathbf{F}_f \mathbf{F}^{-1}] \phi^s + \phi_f^s \equiv \mathbf{T} \phi^s + \phi_f^s, \quad (2.22)$$

for a *hard* target (with velocity being 0 on the boundary); or

$$\phi_f = [-\mathbf{G}_f \mathbf{G}^{-1}] \phi^s + \phi_f^s \equiv \mathbf{T} \phi^s + \phi_f^s, \quad (2.23)$$

for a *soft* target (with pressure being 0 on the boundary). The acoustic transfer matrix  $\mathbf{T}$  is computed, stored, and re-used in the same way as in a radiation problem.

## 2.5 Acoustic Panel Contribution Analysis

For radiation problems ( $\phi^s(\mathbf{x}) = 0$  in Eq. (2.14)), it is helpful sometimes to conduct the so called *panel contribution analysis* in order to identify how each panel (a surface or a group of elements) on the vibrating structure contributes to the total sound pressure at a field point at different frequencies. To do this, we first rewrite Eq. (2.14) as follows:

$$\begin{aligned}\phi(\mathbf{x}) &= \sum_{k=1}^{\text{No. Panels}} \phi_k(\mathbf{x}), \\ \phi_k(\mathbf{x}) &= \int_{P_k} [G(\mathbf{x}, \mathbf{y}, \omega)q(\mathbf{y}) - F(\mathbf{x}, \mathbf{y}, \omega)\phi(\mathbf{y})] dS(\mathbf{y}), \quad \mathbf{x} \in E;\end{aligned}\tag{2.24}$$

where  $P_k$  is a panel with  $\bigcup_{k=1}^{\text{No. Panels}} P_k = S$ . The panel *contribution factor* for panel  $P_k$  is defined as:

$$C_k = \frac{\phi_k \cdot \phi}{|\phi|^2} \equiv \frac{\text{Re}\{\phi_k\} \text{Re}\{\phi\} + \text{Im}\{\phi_k\} \text{Im}\{\phi\}}{|\phi|^2}.\tag{2.25}$$

That is, the contribution factor  $C_k$  for panel  $P_k$  is the projection of the calculated pressure component  $\phi_k$  from panel  $P_k$  to the direction of the total sound pressure  $\phi$ . Note that  $C_k$  is a real number which can be positive (meaning a positive contribution of  $\phi_k$  to  $\phi$ ) or negative (meaning a negative contribution of  $\phi_k$  to  $\phi$ ). This information can be used to guide the optimization of the sound responses for radiation problems. From Eqs. (2.24) and (2.25), we also note that:

$$\sum_{k=1}^{\text{No. Panels}} C_k = 1.\tag{2.26}$$

In *FastBEM Acoustics*®, the panel contribution factors are evaluated at only one field point where the pressure magnitude is maximum, if more than one field points are specified.

## 2.6 Fast Solution Methods

Near  $O(N)$  computational efficiencies can be achieved in solving the BEM systems of equations by applying the fast multipole method or the adaptive cross approximation (ACA) method. Both fast solution options are available in *FastBEM Acoustics*® along with a fast direct solver option based on the conventional BEM.

The main idea of the fast multipole BEM is to apply iterative solvers (such as GMRES) to solve Eq. (2.16) and employ the fast multipole method to accelerate the matrix-vector multiplication ( $\mathbf{A}\mathbf{x}$ ) in each iteration, without forming the entire matrix  $\mathbf{A}$  explicitly. Direct integrations are still needed when the elements are close to the source point, while fast multipole expansions are used for elements that are far away from the source point. For the far field calculations, the node-to-node interactions in the conventional BEM (Figure 2.4 (a)) are replaced by cell-to-cell interactions (Figure 2.4 (b)) by using a hierarchical tree structure of cells containing groups of elements (in Figure 2.4, the dots indicate nodes/cells and lines indicate the interactions needed). This is possible by using the multipole and local expansions of the integrals and some translations in the fast multipole method. More information about the fast multipole BEM applied in the program can be found in Refs. [12, 14-19].

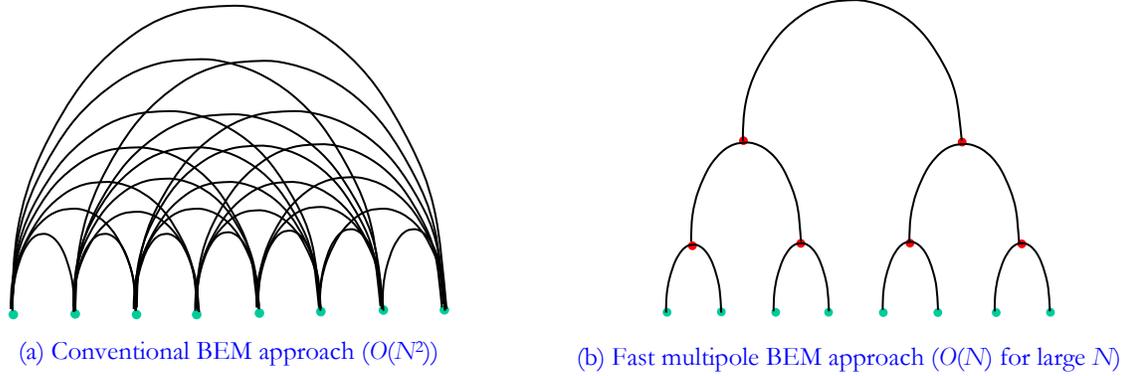


Figure 2.4. A graphical illustration of the conventional BEM and the fast multipole BEM.

Adaptive cross approximation method uses a partition of the matrix in Eq. (2.16) based on the theory of hierarchical matrices and applies the ACA to represent the low-rank submatrices away from the main diagonal. A few row and column vectors are computed and saved for each low-rank submatrix to represent the submatrix. The full matrix is not computed exactly for each coefficient. Therefore, the total memory size for storing the BEM matrix can be reduced. The iterative solver can also be applied to solve the resulting BEM equations to speed up the solution. Compared with the fast multipole BEM, the ACA BEM can deliver faster convergence with the iterative equation solver and the solution time is also less sensitive to the frequency. However, ACA BEM consumes more memory space than the fast multipole BEM, and therefore it is limited to solving BEM models with small to medium sizes (for example, with the total number of DOFs up to 100,000 on a PC with 2 GB RAM). More information about the ACA method can be found in Refs. [20-21].

## 2.7 High-Frequency BEM (HFBEM)

When solving *exterior* radiation problems at high frequencies, the high-frequency BEM can be applied to rapidly estimate the sound field in the far field using a fraction of the CPU time as used by the other more “exact” BEM approaches, as the HFBEM does not solve the BEM system of equations. The HFBEM formulation is similar to the well known Rayleigh integral approximation, but it is more general and can provide better approximate solutions than the Rayleigh integral approximation in many real applications [22].

The assumption used in the HFBEM is that when the structure vibrates at a high frequency, sound pressure on each element on the surface can be determined by the normal velocity directly using the following relation:

$$\phi = -\rho c v_n, \quad \text{on } S, \quad (2.27)$$

in which the “-” sign indicates that the velocity is in the direction of normal  $n$  (Figure 2.1), which is in the opposite direction of the pressure load in our convention. Substituting this result and  $q = i\omega p v_n$  in Eq. (2.14) for a far field point, we have the following *high-frequency approximation* for the sound pressure:

$$\phi(\mathbf{x}) = i\omega\rho \int_S \left[ 1 + \left( 1 - \frac{1}{ikr} \right) (\hat{\mathbf{r}} \cdot \hat{\mathbf{n}}) \right] G(\mathbf{x}, \mathbf{y}, \omega) v_n(\mathbf{y}) dS(\mathbf{y}), \quad \forall \mathbf{x} \in E, \quad (2.28)$$

where  $(\hat{\mathbf{r}} \cdot \hat{\mathbf{n}})$  indicates the dot product of the two unit vectors along  $r$  and normal  $n$  (Figure 2.1). Therefore, the far field pressure can be evaluated directly using this integral when the normal velocity  $v_n$  is given everywhere on the surface  $S$  for a radiation problem, without having to solve the BEM system of equations. If we assume that  $r$  is large and  $(\hat{\mathbf{r}} \cdot \hat{\mathbf{n}}) = 1$  for “visible” elements (can be seen from the field point), then Eq. (2.28) is further reduced to the following integral:

$$\phi(\mathbf{x}) = 2i\omega\rho \int_S G(\mathbf{x}, \mathbf{y}, \omega) v_n(\mathbf{y}) dS(\mathbf{y}), \quad \forall \mathbf{x} \in E, \quad (2.29)$$

which is evaluated on “visible” elements only. This expression is simply the Rayleigh integral approximation [22].

In **FastBEM Acoustics**<sup>®</sup>, the high-frequency approximation, Eq. (2.28), is implemented in the high-frequency BEM for exterior radiation problems (*Solver Type* = 4). Good results of far field sound fields can be obtained very rapidly for exterior radiation problems of vibrating structures with many large flat surfaces [22].

# Chapter 3. Interface with Other CAE Software

Interfacing of the *FastBEM Acoustics*® software with ANSYS® and Nastran, two popular FEA software packages, is currently available. One can build the BEM models using either ANSYS® or MSC/Patran® (a pre- and post-processor for Nastran) and then use the translators to generate the BEM *input.dat* files. The two translator codes can be found in the folder *Interfaces*. The current translator codes can translate the boundary data and applied velocity BC data. More features, such as translating the FEA vibration analysis results, will be added to these two translators in the future.

## 3.1 Interface with ANSYS®

A translator is included with the *FastBEM Acoustics*® package (in the installation folder **C:\Program Files\FastBEM\FastBEM Acoustics**), which can be used to build your own BEM model in ANSYS® and then generate the *input.dat* file for the BEM program. The following steps should be followed strictly in building the BEM models in ANSYS® and generating the BEM *input.dat* file using the translator:

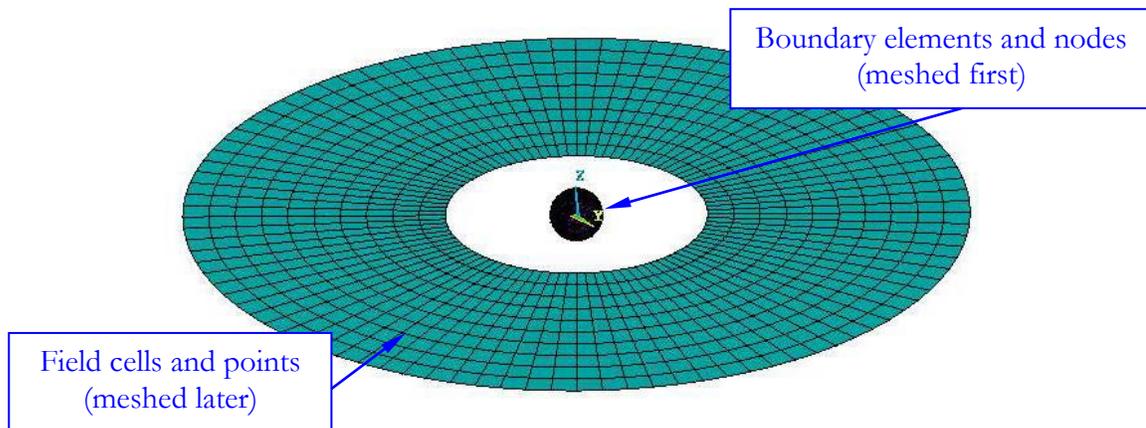


Figure 3.1. Building the BEM model in ANSYS® with the translator.

1. Mesh the surfaces of the structure/object using shell elements (e.g., Shell63) and make sure the normals of all the elements are consistent (e.g., pointing into the structure for exterior acoustic problems). For radiation problems, you can apply a “pressure” load in ANSYS® on the related surfaces to provide the normal velocity values (the translator will take a “pressure” load on an element as the normal velocity for that element in the BEM model). Compress all the nodes and elements so that there will be no gaps in the numbers of these nodes and elements on the boundary (Figure 3.1).

2. If needed, create the field surface and mesh it using Shell63. Select all the nodes on the field surface and create a component to be called “FieldPoints”. Similarly, select all the elements on the field surface and create another component to be called “FieldCells”.
3. Save the ANSYS® model as an archive file using the following commands in ANSYS®:

**Preprocessor/Archive Model/Write**

and choose the option “DB” (All finite element information) and give the file name as “*ANSYS\_Model.cdb*” and select the folder where the translator is located. An ANSYS® archive file *ANSYS\_Model.cdb* will be created.

4. To run the translator program under Windows®, simply click **ansys\_bem\_acoustics.exe** and input the path to the folder where the file *ANSYS\_Model.cdb* is located, or launch the program in a terminal under Linux. You can run this program by clicking the batch file **ansys\_bem\_acoustics.bat** which comes with the package for Windows®, if the input file *ANSYS\_Model.cdb* is located in the same folder (the current folder “.”) as the program file. The ANSYS® archive file *ANSYS\_Model.cdb* will be processed and a BEM *input.dat* file will be created. Edit the parameters (Title, frequency range, etc.) as needed in the *input.dat* file before submitting the BEM job. An *input.plt* file is also generated by the translator code that can be used with Tecplot® to check the model.
5. If you use the GUI **FastBEM View**®, you can launch the translator program in the **File/Import Model/From ANSYS Archive** menu or from the quick tool bar. Select the folder containing the ANSYS® archive file *ANSYS\_Model.cdb* and the BEM model will be imported and displayed in the GUI graphics window. Make proper changes in the **Parameters** and **Solution Options** menus before start the BEM solver.

Note that any Q4 (quadrilateral 4-node) elements on the boundary will be divided into two T3 (triangular 3-node) elements by the translator so that there are only T3 elements on the boundary, as required by this fast multipole BEM program. Field cells (in Q4) will be unchanged by the translator.

## 3.2 Interface with Nastran

A translator is included with the **FastBEM Acoustics**® package (in the installation folder **C:\Program Files\FastBEM\FastBEM Acoustics**) for preparing the *input.dat* file for the BEM program based on the Nastran bulk data for a model. Nastran bulk data can be prepared using many CAE pre- and post-processing software tools, such as MSC/Patran® and Altair HyperWorks®. The following steps need to be followed in building the BEM models using the Nastran bulk data:

1. Mesh the surfaces of the structure/object using Q4 (CQUAD4) or T3 (CTRIA3) elements and make sure the normal of all the elements are consistent (e.g., pointing into the structure for exterior acoustic problems). For radiation problems, you can apply a “pressure” load (PLOAD4) on the related elements to provide the normal velocity values (the translator will take a “pressure” load on an element as the normal velocity for that element in the BEM model). Renumber all the nodes and elements so that there will be no gaps in the

- numbers of these nodes and elements on the boundary (Figure 3.2). Use the value “1” as the property identification number (PID) for all the boundary elements.
- If needed, create the field surface and mesh it using Q4 (CQUAD4) elements. Use the value “2” as the PID for all the elements on the field surface.
  - Save the Nastran model as a bulk data file and choose the file name as “*Nastran\_Model.blk*”.
  - To run the translator program under Windows® directly, simply click **nastran\_bem\_acoustics.exe** and input the path of the folder where the Nastran bulk data file *Nastran\_Model.blk* is located, or launch the program in a terminal under Linux. You can run this program by clicking the batch file **nastran\_bem\_acoustics.bat** which comes with the package for Windows®, if the input file *ANSYS\_Model.cdb* is located in the same folder (the current folder “.”) as the program file. The Nastran bulk data file *Nastran\_Model.blk* will be processed and a BEM *input.dat* file will be created. Edit the parameters (Title, frequency range, etc.) as needed in the *input.dat* file before submitting the BEM job. An *input.plt* file is also generated by the translator code that can be used with Tecplot® to check the model.
  - If you use the GUI **FastBEM View**®, you can launch the translator program from the **File/Import Model/From Nastran Bulk Data** menu or from the quick tool bar. Select the folder containing the Nastran bulk data file *Nastran\_Model.blk* and the BEM model will be imported and displayed in the GUI graphics window. Make proper changes in the **Parameters** and **Solution Options** menus before start the BEM solver.

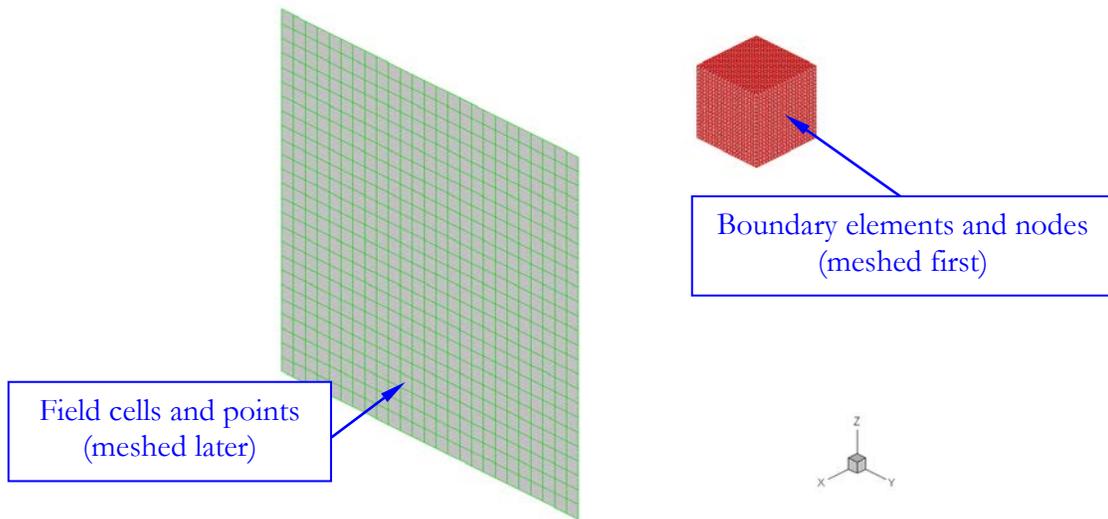


Figure 3.2. A BEM model of a box translated from Nastran bulk data.

Note that only the fixed format for the bulk data entries can be used with the translator. Free format of the bulk data is not supported and will produce errors when it is used with the translator.

# Chapter 4. Modeling and Solution Considerations

Discretization of the boundary using elements and selection of the parameters specified in *input.dat* model file and *input.fmm* parameter file can have significant impact to the performance of the program, regarding both accuracy and efficiency of the BEM solution. Careful considerations in the discretization and selection of these parameters are necessary in order to achieve the optimal efficiency in solving a large-scale BEM model while keeping the errors in control. Some of the considerations are discussed in the following sections.

## 4.1 Discretization

As a rule of thumb, the number of elements in a BEM model should be sufficiently large such that there are at least 6-10 elements per wavelength  $\lambda$  which can be calculated using

$$\lambda = c / f = 2\pi / k \quad (4.1)$$

at a given frequency  $f$ . The above formula can be used to estimate the required element size in the model. The shape of the elements can also affect the conditioning of the BEM system of equations, which in turn will determine the number of iterations needed to solve the system using iterative solvers. For better conditioning and better efficiency in the BEM solution, the elements in a BEM mesh should be as close as possible to an equilateral triangle (as the one shown in [Figure 2.3](#)). Seriously distorted elements can lead to higher condition number and cause slow convergence.

## 4.2 Use of the Dual BIE

The dual BIE should be applied *only when it is necessary*, for example, in cases when a fictitious eigenfrequency (irregular frequency) is encountered for an exterior problem, or when the domain contains very thin shapes (shell-like structures). To use the dual BIE, give the value of 1 for the HBIE in the *input.dat* file. The program will automatically determine the value of the coupling constant  $\beta$  in Eq. (2.13). The CBIE formulation in Eq. (2.8) in general gives better results and is more efficient when it is applied at a regular frequency or in a domain with no thin shapes.

## 4.3 Parameters *nruleb* and *nrulef*

Parameter *nruleb*, which has the values of 1, 2, 3, 4, 5 and 6, determines the integration quadrature rule or the number of Gaussian points (see [Table 4.1](#)) in the numerical integration on a triangle used for evaluating the boundary integrals. Large numbers of Gaussian points can improve the accuracy, but can also reduce the efficiency of the code. For most problems, selection of *nruleb* = 3

has been found to be sufficient for the accuracy of the BEM results.

Table 4.1. Integration quadrature rule and the corresponding number of Gaussian points.

| <i>Integration Quadrature Rule</i><br>( <i>nruleb</i> or <i>nrulef</i> ) | 1 | 2 | 3 | 4 | 5  | 6  |
|--|---|---|---|---|----|----|
| <i>Number of Gaussian Points</i>   | 1 | 3 | 6 | 9 | 16 | 64 |

Parameter *nrulef*, which also has the values of 1, 2, 3, 4, 5 and 6, specifies the integration quadrature rule for the field points. If the field points are far away from the boundary of the structure, *nrulef* = 1 (using a one-point quadrature) has been found to be sufficient for the accuracy. For field points that are close to the boundary, a larger value for *nrulef* should be selected.

#### 4.4 Parameter *maxl*

Parameter *maxl* determines the maximum number of elements allowed in a leaf in the tree structure for the fast multipole BEM, which also determines the size of the submatrices used as the preconditioner in solving the BEM model using the iterative solver. Larger values of *maxl* also means use of larger memory (RAM) size since the preconditioner is saved in the solution process if parameter *nstke* is non-zero. On a PC with a limited RAM, a range of 20-100 for *maxl* has been found to be efficient for the code; while on a supercomputer having a larger size of RAM, a range of 100-500 for *maxl* has been found to be more efficient.

#### 4.5 Parameter *nexp* (*p*)

Parameter *p*, which is the maximum value for the order of the multipole expansions to be used for the fast multipole BEM, can have significant effects on the accuracy and efficiency of the solutions. While higher values of *p* can give better accuracy, the solution time can increase quickly. In general, as a rule of thumb, the following empirical formula can be used in estimating the value for *p*:

$$p = ka / 4, \text{ up to } p = 24 \quad (4.2)$$

where *k* is the wavenumber and *a* the largest dimension of the model. Higher values of *p* can be selected if higher accuracy is required. The adequate range of the values of *p* has been found to be from 4 to 24 for most problems. The program will use a more sophisticated algorithm to automatically estimate a value of *p* on each cell in the tree structure depending on the size of the tree structure and the frequency (that is, using an adaptive *p*), based on the user specified value for *p*. Larger values of *p* (e.g., with *p* > 24) may cause data overflow due to the small values involved in the expansions for some models. When this happens, simply select a smaller value for *p*.

#### 4.6 Parameter *tolerance* ( $\epsilon$ )

For most engineering applications, numerical results with 3 or 4 significant digits may be sufficient, especially when plotting these results in the form of color contour plots. Therefore, a selection of

the tolerance  $\varepsilon = 10^{-3}$  or  $10^{-4}$  for the convergence of the solutions with the iterative solver should be sufficient, unless the users wish to test the code with more stringent criteria as in the case of comparing the BEM results with analytical solutions.

## 4.7 Parameter *nstk*

Parameter *nstk* is related to the memory size allocated to store the pre-conditioners. Saving these pre-conditioners can significantly increase the efficiency. The default value of *nstk* = 200000000 is adequate for a PC with 16 GB RAM. This value of *nstk* can be adjusted, for example, be further increased if larger models need to be solved and more RAM is available on the computer. For example, one can use *nstk* = 500000000 to 800000000 for a PC with 32 GB or more RAM and significant reductions in the solution time can be achieved.

## 4.8 Use of FMM for Field Evaluation (*ifmm*)

The field evaluation can be done in two ways: Using the direct integration which is more accurate, or the fast multipole method which is more efficient for large models and with a large number of field points. To use the FMM for field evaluations, change the value of the parameter *ifmm* to 1 in file *input.fmm* and select a value for the expansion order *nexpf*. To obtain reasonably accurate results for the field evaluations using the FMM, smaller values for *maxl* (number of elements per leaf cell) and larger values of *nexpf* may be needed if the field surface is far away from the boundary surface of the domain.

## 4.9 Solver Type

There are four BEM solvers available in the program, namely, the fast multipole BEM, the ACA BEM, the conventional BEM, and the high-frequency BEM. The fast multipole BEM with the iterative linear equation solver is the most efficient BEM solver for solving large-scale BEM models (for example, with the number of DOFs above 100,000, depending on the RAM size and the OS of the computer). However, its performance may slow down for solving models at high frequencies (when the non-dimensional wavenumber *ka* is, for example, above 200, for which a large value of expansion order *p* is required). The ACA BEM also uses iterative equation solver and expansions of the coefficients. However, for small to medium sized models, it can be more efficient than the fast multipole BEM. Also, its efficiency is not very sensitive to the frequency. The drawback of the ACA BEM is that it requires a larger RAM size as compared with the fast multipole BEM. The conventional BEM uses a fast direct linear equation solver and delivers the most accurate result. However, it is limited to solving smaller BEM model due to its requirement of large memory size and long CPU time.

It is suggested that users select the fast multipole BEM first as the solver type, and then select the ACA BEM or the conventional BEM, if the preceding selection does not deliver satisfactory results. This is also the order that the program will select by default (for example, switch to the ACA BEM, if the fast multipole BEM fails to converge within the specified maximum number of iterations). The high-frequency BEM is an approximation and should be applied when a fast

estimate of the sound pressure is needed for exterior radiation problems.

## 4.10 Use of Complex Wavenumber

**FastBEM Acoustics**® allows users to specify a complex wavenumber  $k_c$  in order to introduce dissipation, attenuation or damping effect in the acoustic medium. This is done by specifying a value of a new parameter called wavenumber  $k_{ratio}$  on the data line for the properties of the acoustic medium in *input.dat* file. The complex wavenumber  $k_c$  is defined as:

$$k_c = k(1 + i k_{ratio}) \quad (4.3)$$

where  $k_{ratio}$  is the  $k$  ratio parameter whose value is determined from the *input.dat* file, and  $k$  is still defined as  $k = \omega / c$ . That is,  $k_{ratio}$  is defined as the ratio of the imaginary part to real part of the complex wavenumber. Note that when  $k_{ratio} = 0$ , we have  $k_c = k$  as before (no damping effect).

A complex wavenumber can come from different sources and in different forms [6]. How to input the value of  $k_{ratio}$  will be case dependent if it is to be used. For example, if we have an acoustic medium producing decaying waves and with a complex speed of sound  $c_c$  in the form

$$c_c = c_r - i c_i \quad (4.4)$$

with  $c_r > 0$  and  $c_i \geq 0$ , then this  $c_c$  value cannot be specified to the program directly. In this case, we can use  $k_{ratio}$  for this purpose. We have

$$k_c = \frac{\omega}{c_c} = \frac{\omega}{c_r - i c_i} = \frac{\omega}{c_r^2 + c_i^2} (c_r + i c_i) = \frac{\omega}{(c_r^2 + c_i^2) / c_r} \left( 1 + i \frac{c_i}{c_r} \right) = \frac{\omega}{c} (1 + i k_{ratio}) \quad (4.5)$$

and use the following input for  $c$  and  $k_{ratio}$  to run the program:

$$c = (c_r^2 + c_i^2) / c_r, \quad k_{ratio} = c_i / c_r \quad (4.6)$$

Note that in general, the value of  $k_{ratio}$  is much smaller compared to 1. In most cases, one can set  $k_{ratio} = 0$ , if no damping effect or decaying waves need to be considered.

The use of a complex wavenumber can also speed up the solutions using the fast multipole BEM and ACA BEM solver options, besides allowing users to model the effects of dissipation or attenuation or decaying of acoustic waves. The introduction of the damping effect into the system can smooth the response curve (by reducing the magnitudes of the peaks) and therefore obtain faster convergence with iterative solvers. It is found that use of an artificial value of  $k_{ratio} = 0.01$  to 0.03 can cut the number of iterations by more than one half with the iterative solver in many cases, especially at higher frequencies.

Examples using complex wavenumbers can be found at the website [www.fastbem.com](http://www.fastbem.com) and on the page *Verification Cases*.

## 4.11 Parallel Computing (*No. Threads*)

**FastBEM Acoustics**® parallel version is parallelized using OpenMP® which can use multi-cores and threads of the CPU(s) to speed up the computation in solving a BEM model. To use this parallel computing feature, set the parameter *No. Threads* > 1 on the second line in the *input.dat* file. If *No. Threads* is set to 0, specified incorrectly, or not specified, then one half of all the threads of the CPU(s) of the computer will be used by default in solving the BEM model. To run the code in sequential mode (using a single thread), set *No. Threads* = 1.

To find out the number of threads for the CPU(s) on your computer, check the specification of the CPU(s) (For Intel® processors, check the website <http://ark.intel.com/>). For most Intel® multi-core CPUs, each core has two threads. Therefore, for example, an Intel® CPU with four cores will have eight threads that can be used for the parallel computing. However, it is found that the use of all the threads is not very efficient and can actually slow down the computation. Use of one half of the total threads will be a more efficient choice (or leave at least one or two threads for other processes running on the CPU(s)).

# Chapter 5. Examples

In this chapter, a sphere model and a box model are used to show how to run the *FastBEM Acoustics*® program and how to process the obtained BEM results using Tecplot® (For processing the BEM results using the GUI *FastBEM View*®, see the separate *FastBEM View Guide*). Both full-space and half-space problems will be demonstrated.

## 5.1 Scattering from A Rigid Sphere

A sphere scattering problem is considered first. The sphere has a radius  $R = 1$  and a total of 1200 elements are used in this case. The parameters used in *input.dat* file (lines 4-10) are as follows:

```

1200   602  1680  1600   1  ! Nos. of Boundary Elements/Nodes, and Nos. of Field Points/Cells, No. Panels
1       0                ! No. of plane waves (nplane); ; User defined sources (1=Yes/0=No)
(1., 0.) -1.  0.  0.      ! Complex amplitude and direction vector of the plane wave(s)
0       0                ! No. of monopole sources, and no. of dipole sources
343.00  1.29  2.d-5  1.d-12  0. ! cspeed (c), density (rou), p_ref, I_ref, k ratio
54.59  54.59  1  0  0  ! Freq1, Freq2, No. freqs, NOctave, Update BC/freq (1=Yes/0=No)
0       3  1  0  One  ! HBIE (1=Yes/0=No), nruleb, mulef, animation (1=Yes/0=No), Tecplot Data (All/One)

```

That is, an incident wave is coming along the  $-x$  direction, and the CBIE is applied to solve the model at a single frequency with  $ka = 2$  ( $k = 2\pi f / c$  and  $a = 2R = 2$  m). The default values of the parameters in *input.fmm* file are used for the solution.

After the model is solved, results in *output\_tecplot.plt* file are used with Tecplot® to plot the results on the boundary of the sphere (Figure 5.1), field surface (Figure 5.2), or both (Figure 5.3).

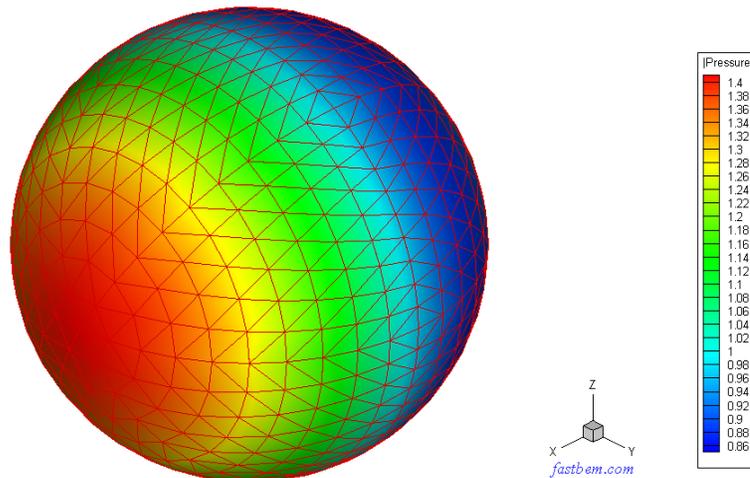


Figure 5.1. BEM mesh and pressure plot on the sphere (with 1200 boundary elements).

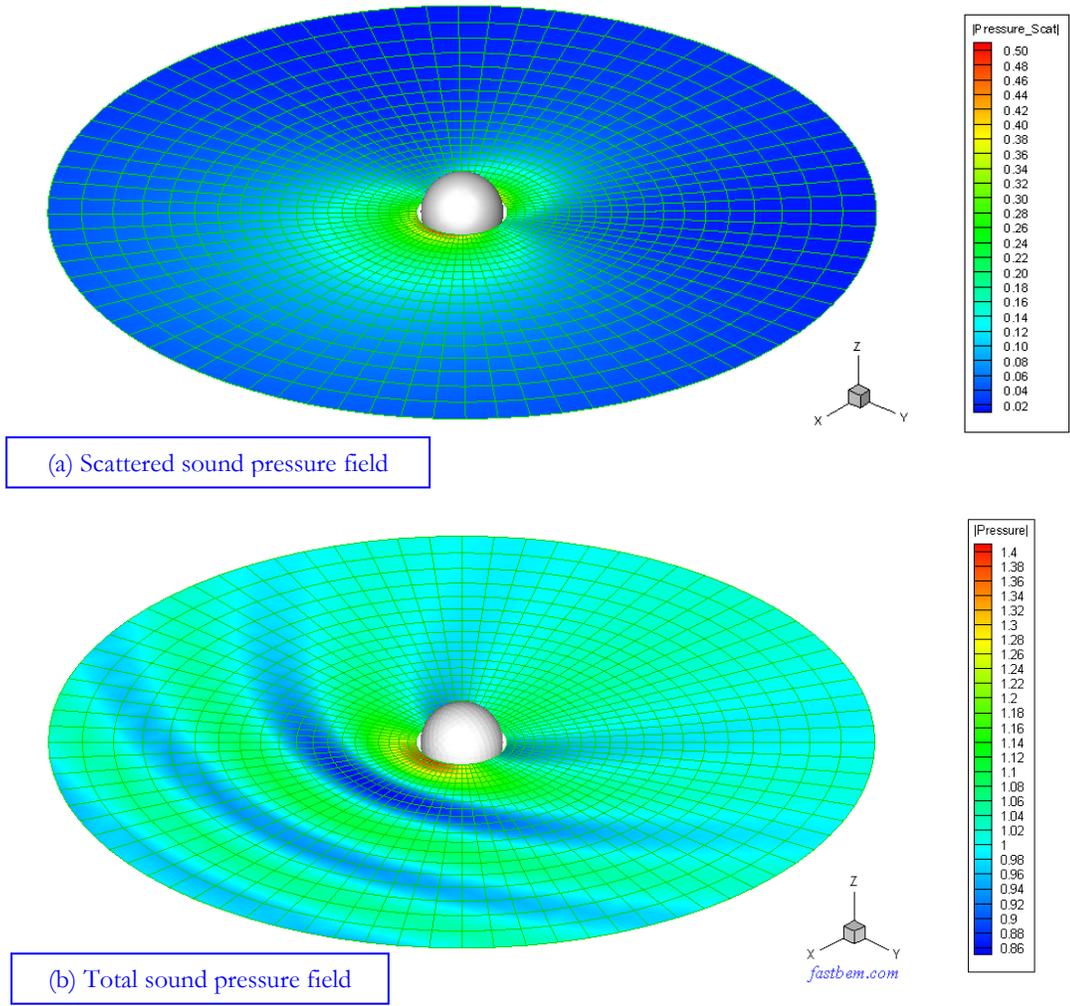


Figure 5.2. Plot of sound pressure on the field surface (with 1600 field cells for plotting).

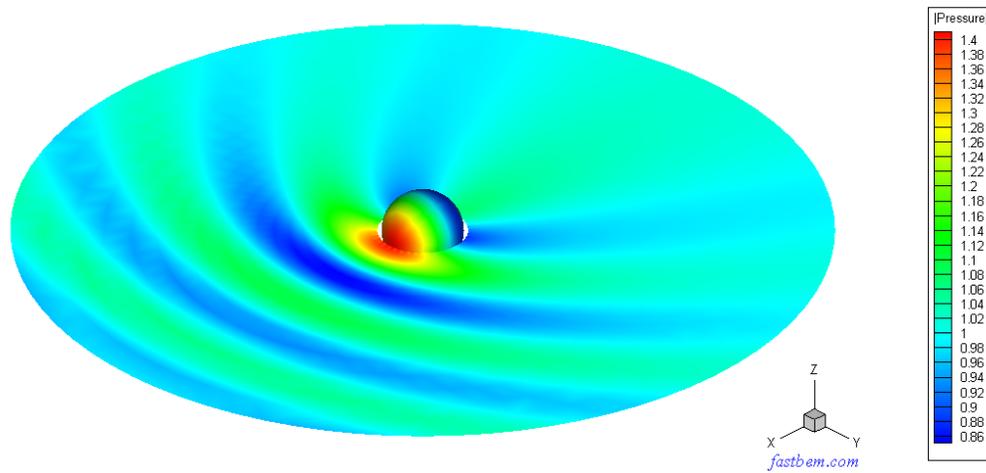


Figure 5.3. Plot of total pressure on both the boundary and field surface (at  $ka = 2$ ).

## 5.2 Radiation from A Pulsating Sphere

In this case, a pulsating sphere model is considered. It is assumed that the sphere of radius  $R = 1$  m is vibrating in the radial direction with a surface velocity of 1 m/s. The parameters used in *input.dat* file (lines 4-9) read as follows:

```

1200   602 1680 1600   1 !Nos. of Boundary Elements/Nodes, and Nos. of Field Points/Cells, No. Panels
0      0      !No. of plane incident waves (nplane); User defined sources (1=Yes/0=No)
0      0      !No. of monopole sources, and no. of dipole sources
343.00 1.29 2.d-5 1.d-12 0. !cspeed (c), density (rou), p_ref, I_ref, k ratio
545.9  545.9 1    0    0 !Freq1, Freq2, No. freqs, NOctave, Update BC/freq (1=Yes/0=No)
0      3      1    0    One !HBIE (1=Yes/0=No), nruleb, nrulef, animation (1=Yes/0=No), Tecplot Data (All/One)

```

and a velocity BC  $v_n = (-1., 0.)$  is applied at all the nodes of the BEM model. That is, the pulsating sphere model at the frequency 545.9 Hz ( $\kappa a = 20$ ) needs to be solved. The number of multipole expansion terms is set at 10 in parameter file *input.fmm*.

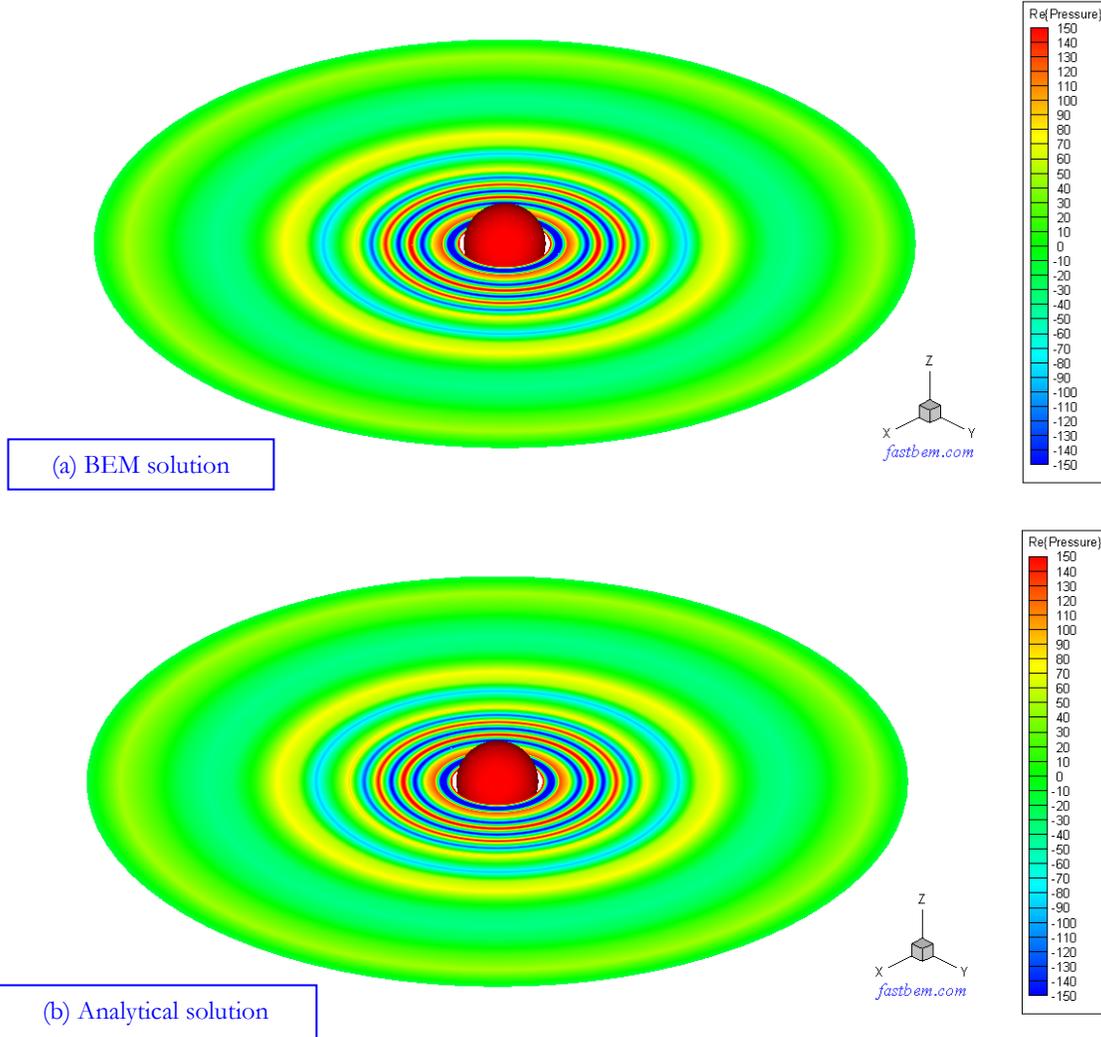


Figure 5.4. Comparison of the computed sound pressure on the field surface at  $\kappa a = 20$ .

The computed sound pressure and sound pressure level (SPL) are plotted in Figure 5.4 and Figure 5.5, respectively, and compared with the analytical solutions. These BEM solutions are indistinguishable from the analytical solutions in these contour plots.

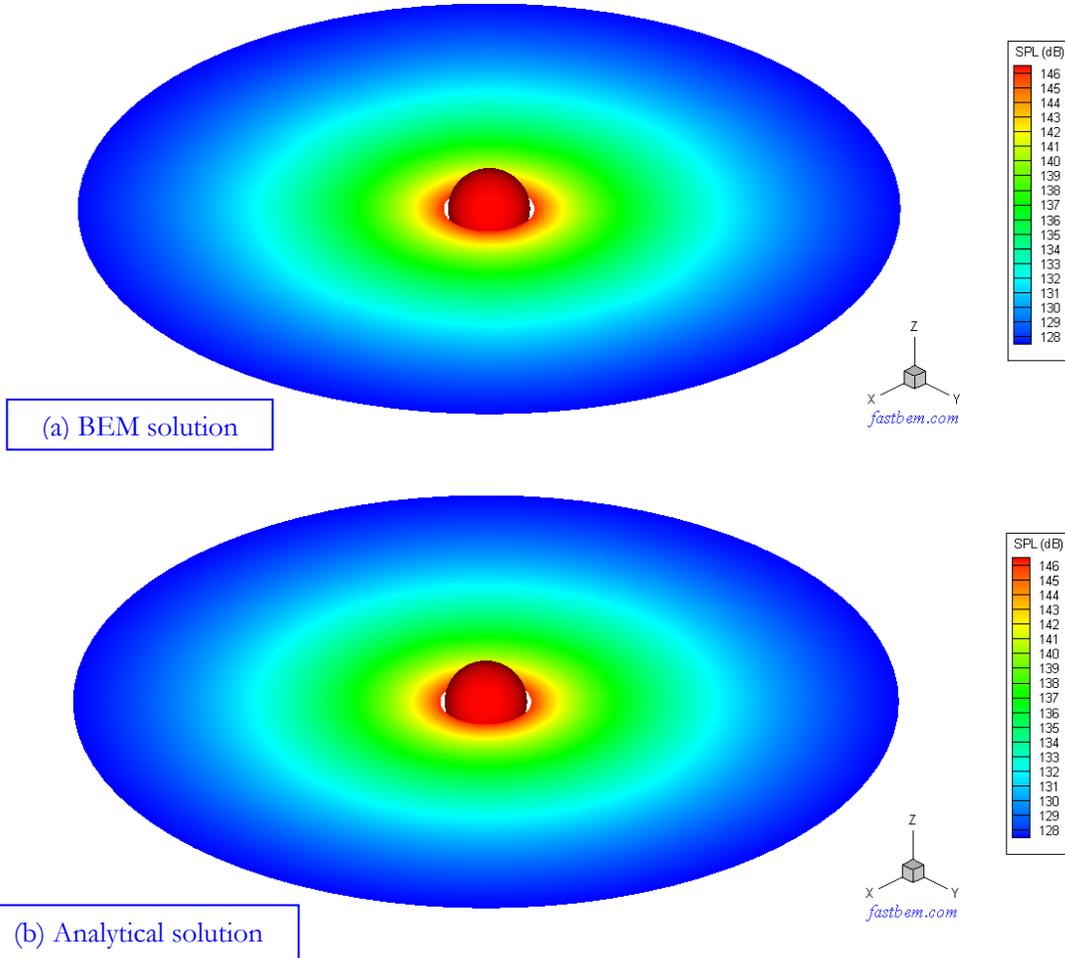


Figure 5.5. Comparison of the computed sound pressure level (SPL) on the field surface at  $ka = 20$ .

### 5.3 Frequency Response Analysis

In this example, multiple frequency (*frequency sweep*) analysis and use of the data in file *output\_freq\_responses.plt* to plot the frequency response curves are demonstrated.

First, the pulsating sphere is discretized using a mesh with 10,800 elements and the frequency range is set to be from 20 Hz to 1000 Hz with a uniform frequency step of 20 Hz (50 frequencies). The line in *input.dat* file defining the frequency values reads as follows:

```
20. 1000. 50 0 0 !Freq1, Freq2, No. freqs, NOctave, Update BC/freq (1=Yes/0=No)
```

The quadrature rule for boundary integration is set at  $nruleb = 6$  in model file *input.dat* and the multipole expansion order is set at  $p = 12$  in parameter file *input.fmm*.

Using the data in the output file *output\_freq\_responses.plt*, several frequency response curves can be generated using any plotting software. Figures 5.6 shows plots of the computed amplitude of the sound pressure and sound pressure level (SPL) (at a point on the field surface that has a distance of  $5R$  from the center of the sphere), the radiated sound power and sound power level (PWL) of the pulsating sphere. Both the CBIE and dual BIE are applied and the BEM results are compared with the analytical solutions. The results using the CBIE do deteriorate at several frequencies, indicating near the fictitious eigenfrequencies/irregular frequencies for the CBIE that occur more frequently at higher frequencies. The results using the dual BIE exhibit smooth variations and in general are more consistent than the results using the CBIE.

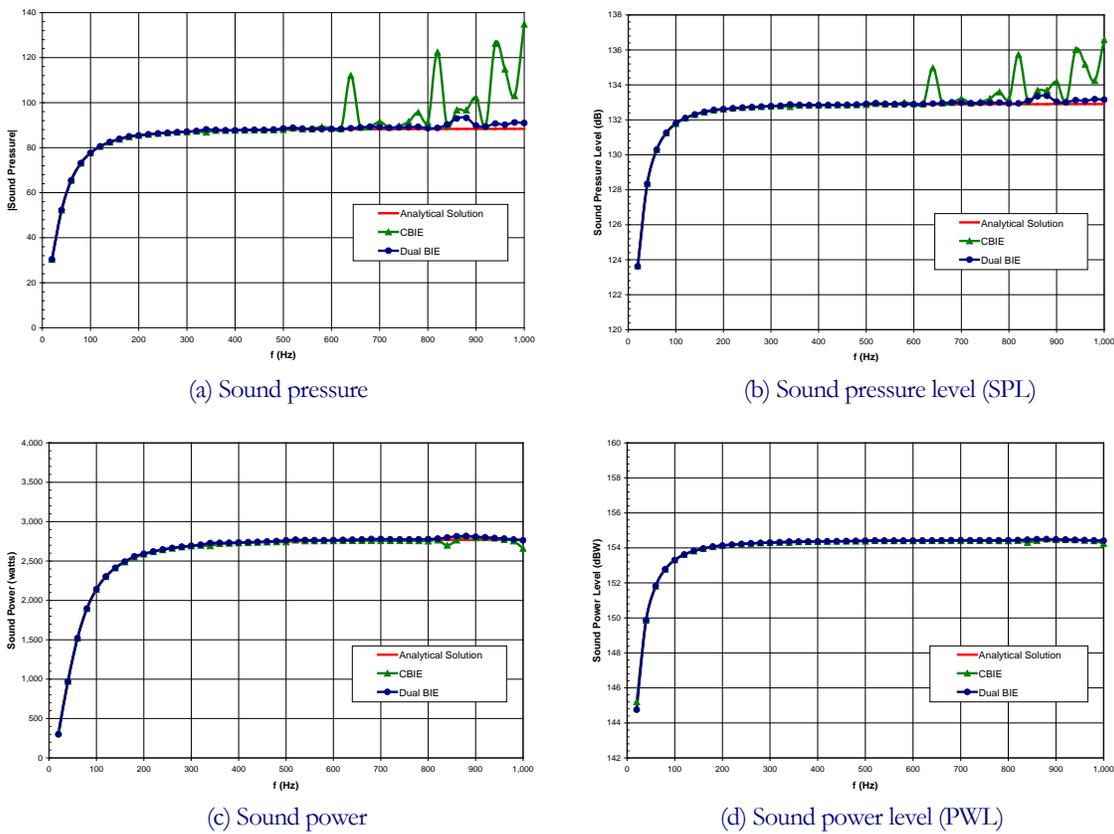


Figure 5.6. Results for frequencies up to 1000 Hz ( $\kappa a = 36.6$ ) using uniform frequency steps.

Next, a multiple frequency case using the 1/3-octave band for frequencies from 20 Hz to 8000 Hz is demonstrated. The line in *input.dat* file defining the frequency values reads as follows for the first frequency range (Four jobs for four ranges are used to compute the results, see Table 5.1):

```
20. 1000. 0 3 0 !Freq1, Freq2, No. freqs, NOctave, Update BC/freq (1=Yes/0=No)
```

The element size and number of elements in the BEM model are determined according to the rule that there should be close to 6–10 elements per wavelength (which is given by

$\lambda = c / f = 2\pi / k$ ) at the corresponding frequency  $f$ .

Table 5.1. BEM models and parameters used for the 1/3-octave band analysis.

| Frequency Range (Hz) | Element Size (m) | Number of Elements | <i>nruleb</i> | <i>p</i> |
|----------------------|------------------|--------------------|---------------|----------|
| 20 – 1000            | 0.05236          | 10,800             | 5             | 12       |
| 1000 – 2000          | 0.02618          | 43,200             | 5             | 24       |
| 2000 – 4000          | 0.01309          | 172,800            | 5             | 24       |
| 4000 – 8000          | 0.00785          | 480,000            | 5             | 24       |

The results in output file *output\_freq\_responses.plt* are used to generate the plots in Figures 5.7, where for the octave band analysis, the output data are in the form of step functions. The BEM solutions are solved at the center of each frequency band by the program. To have a more uniform representation, the log scale is used in the frequency axis in these plots.

Again in this analysis, several irregular frequencies for the CBIE are encountered in the frequency sweep analysis that spans a wider range of frequencies. It is wise to employ the dual BIE in such cases to avoid the incorrect BEM results. On the other hand, the radiated sound power and sound power level can be determined more accurately using the BEM models for this high-frequency problem using either the CBIE or dual BIE formulation, because of the additional integration involved (see Eq. (1.2)).

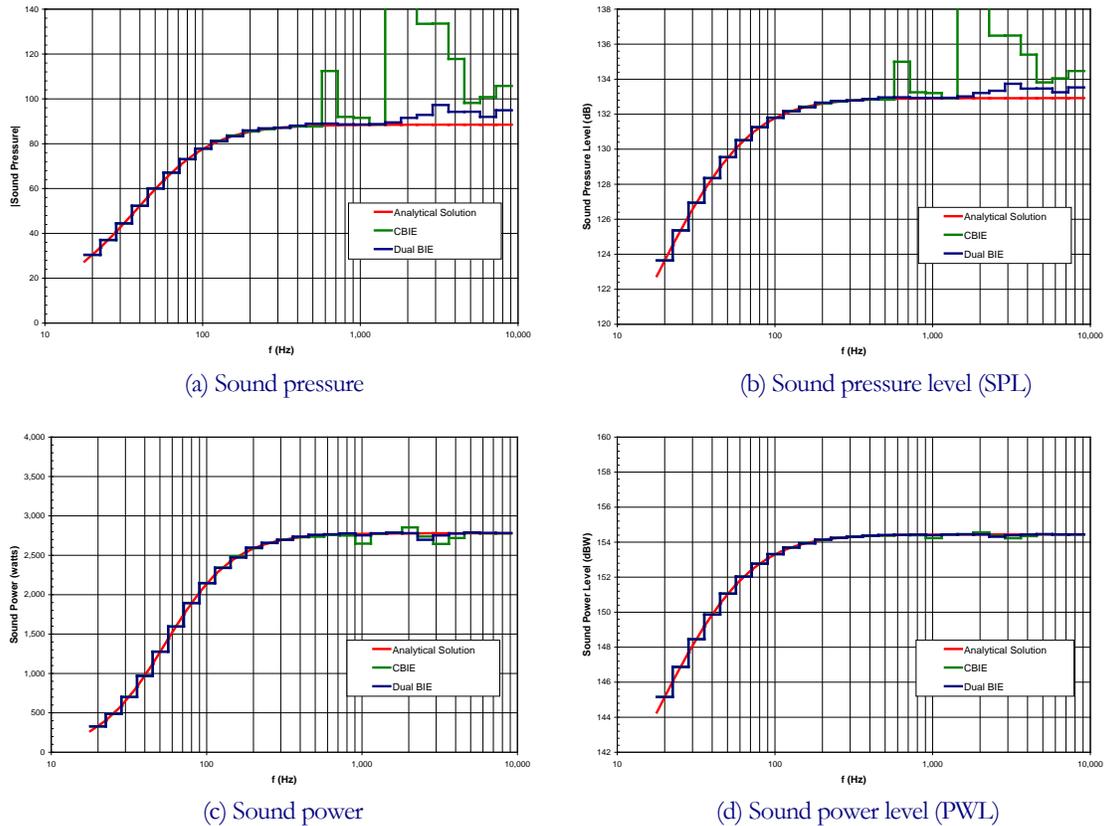


Figure 5.7. Results for frequencies up to 8000 Hz ( $ka = 293$ ) using the 1/3-octave band.

Complete BEM results on all the boundary elements and at all the field points, at all the frequency steps, are available in output file *output\_result.dat*, which can be used to generate other customized plots and/or curves using in-house or other plotting software.

### 5.4 Half-Space Problems

In this example, modeling half-space acoustic problems is demonstrated. A pulsating sphere above an infinite half-space plane on the *x-y* plane is considered. The distance between the sphere and the infinite plane is equal to the radius  $R (= 1)$ . The sphere is discretized with 1200 elements and the wavenumber is  $ka = 2$ . The third line of *input.dat* file reads as follows:

```
Half      3  1.d0      ! Prob. Space, Symm. Plane, Symm. Plane Property (1=Rigid; -1=Soft)
```

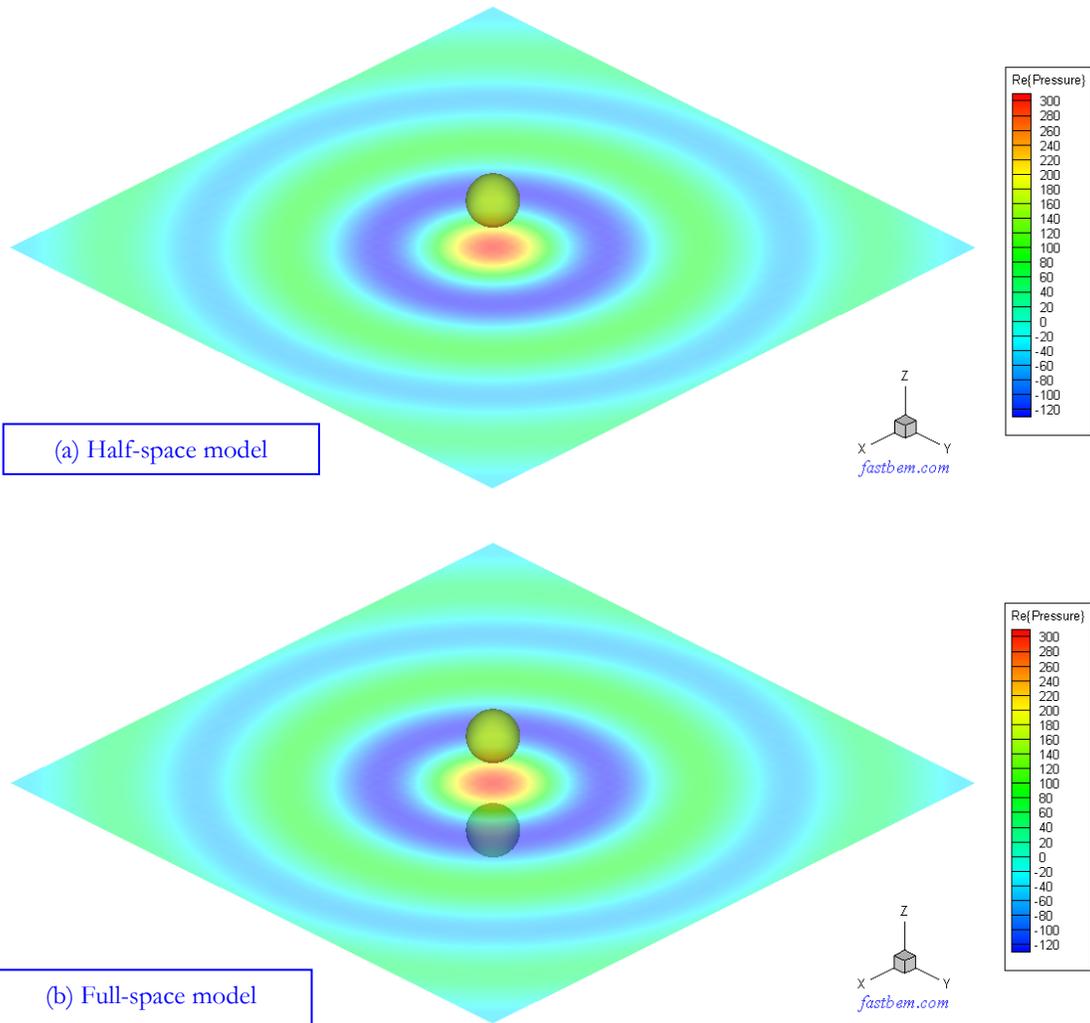


Figure 5.8. Pressure plots for the half-space (with a *rigid* infinite plane) and full-space models.

For comparison, a full-space model with two spheres (one above and another below the  $x$ - $y$  plane) is also considered and is applied with either symmetry BCs (corresponding to a half-space problem with a rigid infinite plane) or antisymmetry BCs (corresponding to a half-space problem with a soft infinite plane). The contour plots of the computed pressure fields for the case with a rigid infinite plane (where the velocity is zero) is shown in Figure 5.8 (in translucent plot in order to show the sphere below the infinite plane in the full-space case). The contour plots of the velocity fields for the case with a soft infinite plane (where the pressure is zero) is shown in Figure 5.9.

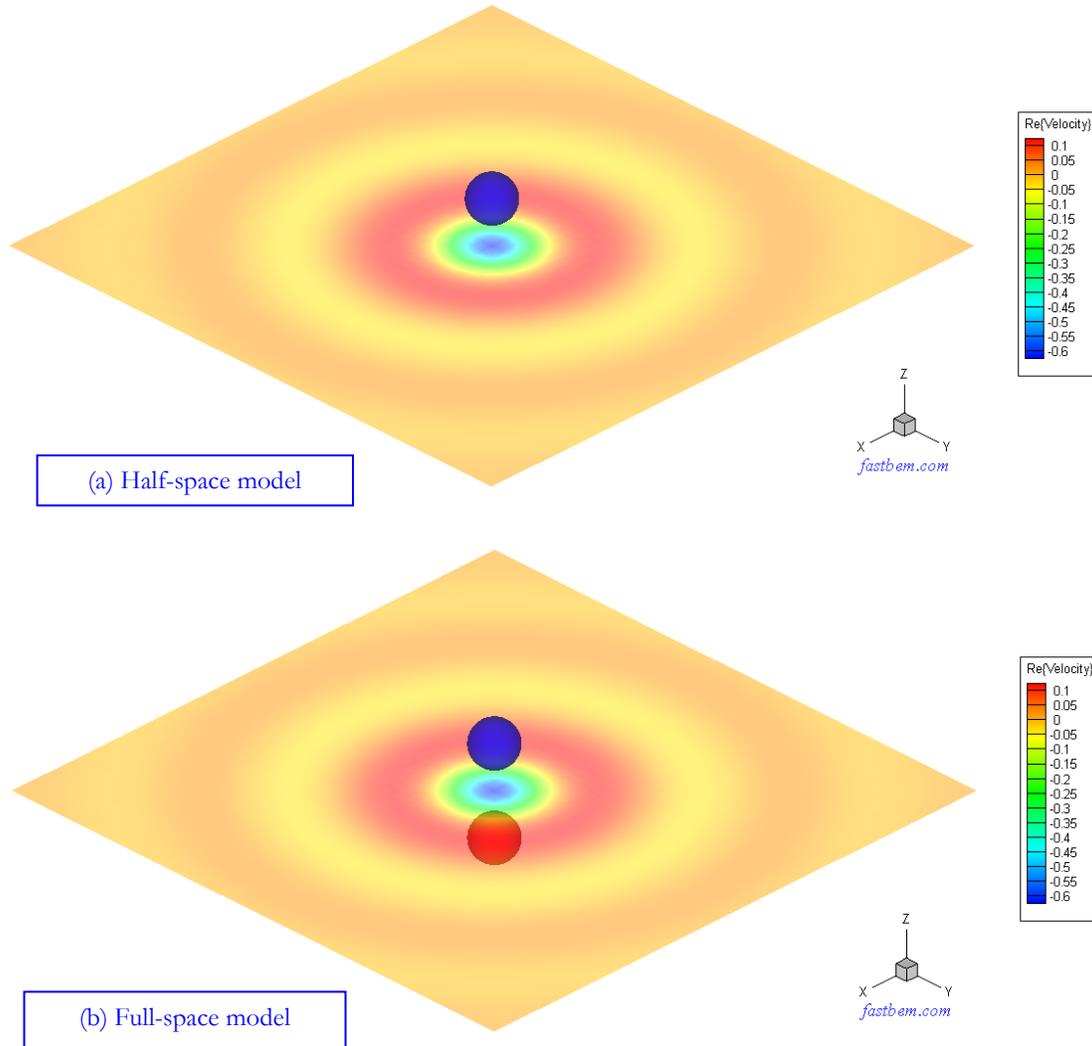


Figure 5.9. Velocity plots for the half-space (with a *soft* infinite plane) and full-space models.

In modeling half-space/symmetry problems, the BEM models can be placed either above or below an infinite half-space/symmetry plane that can be any of the three coordinate planes ( $x$ - $y$ ,  $y$ - $z$ , or  $z$ - $x$  plane, corresponding to the parameter *Symm\_Plane* = 3, 1, 2, respectively). The model can sit or touch the infinite half-space/symmetry plane, but should not go through this plane.

## 5.5 Use of ATM for Scattering from Different Sources

The same sphere scattering problem given in Section 5.1 is considered again using the acoustic transfer matrix (ATM) option for multiple incident waves. All the parameters and model are the same as before, except for the use of the ATM option. First, we generate the ATM with the 2nd line of the *input.dat* file changed to the following:

```
ATM 1 1 ! Job Type (Complete/Field Only/ATM/Use ATM); Solver; No. threads
```

The BEM matrix is computed and stored in the file *output\_ATM\_Saved.dat* and the total pressure on the field surface is evaluated as shown in Figure 5.10 (a) (which is the same as shown in Figure 5.2).

Now, if we want to see the results for a plane incident wave in the -y direction, we can simply use the ATM option, and change the 2nd line of the *input.dat* file to the following:

```
Use ATM 1 1 ! Job Type (Complete/Field Only/ATM/Use ATM); Solver; No. threads
```

and the incident wave direction to (0, -1, 0). The field evaluation is done quickly using the ATM option (using data stored in the file *output\_ATM\_Saved.dat*) and the contour plot of the total sound pressure on the field surface is shown in Figure 5.10 (b).

Similarly, we can obtain the results on the field surface quickly for a monopole source or a dipole source placed, for example, at the location (0, 0, 1.5R), by changing the incident wave to a monopole or dipole in the *input.dat* file and with the *Use ATM* job type option. The contour plots for the total sound pressure on the field surface for these two cases are shown in Figure 5.10 (c) and Figure 5.10 (d), respectively.

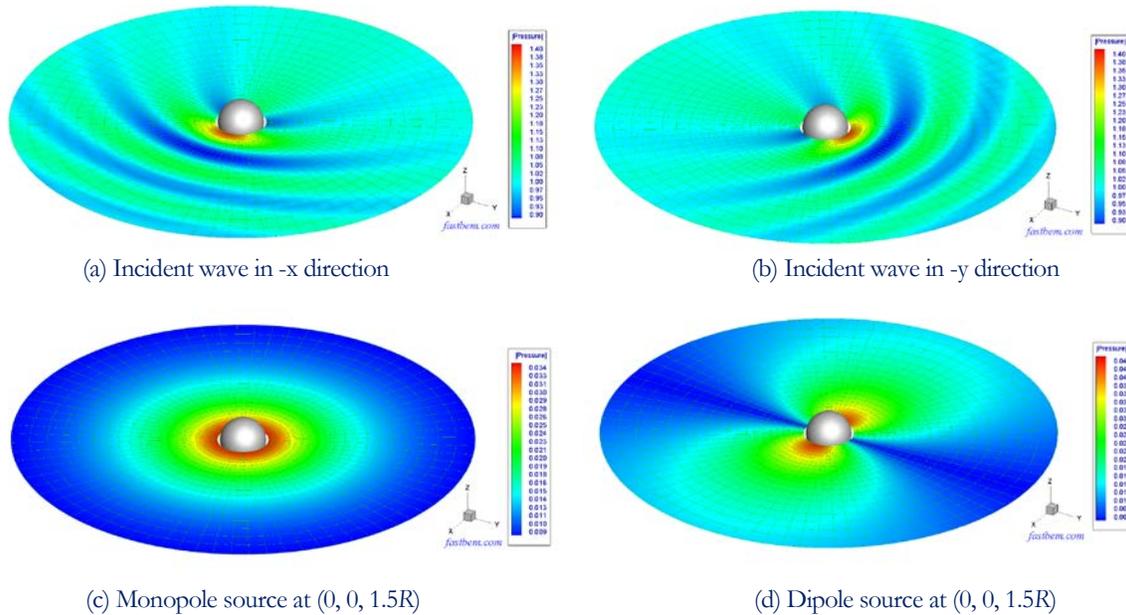


Figure 5.10. Use of the ATM for scattering problems with different incident waves or sources.

Note that only the results on the field surface can be evaluated using the *Use ATM* job type option, results on the structure (scatterer) are not evaluated with this option. Also, for scattering problems with the ATM option, the BC values must be 0 (either a *hard* scatterer or a *soft* scatterer).

Use of the ATM option for radiation problems follows the same procedure. The only caution is that on the entire boundary, only one type of BCs, either the velocity or the pressure (not a mix), can be applied with the *ATM* job type option. In addition, only the values of the boundary condition can be changed in the following re-run with the *Use ATM* job type option.

## 5.6 Panel Contribution Analysis of A Vibrating Box

In this example, the sound field inside a box is studied. The box is located at  $0 \leq x, y, z \leq 1$  m and discretized with 1200 elements. The elements on each of the six faces of the box are grouped into one panel as shown in Figure 5.11. Each element has a normal velocity of 0.001 m/s. The medium inside the box is air and one field point is placed at the location (0.1 m, 0.2 m, 0.3 m).

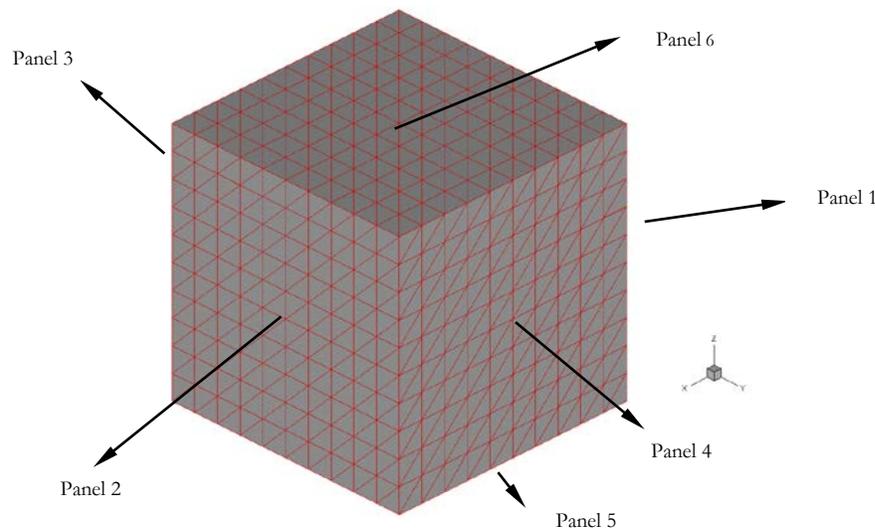


Figure 5.11. A vibrating box with elements grouped into six panels.

Figure 5.12 shows the sound pressure (a) and the panel contribution factors (b) from each panel computed at the field point in the frequency range of 40 Hz – 2000 Hz using the data in the output file *output\_freq\_responses\_panels.plt*. From the panel contribution plot, it is clear that there are strong cancellations of the positive and negative contributions from the six panels in the neighborhoods of the four valleys shown in the sound pressure plot, while there are little or no cancellations of the contributions near the peaks of the sound pressure plot.

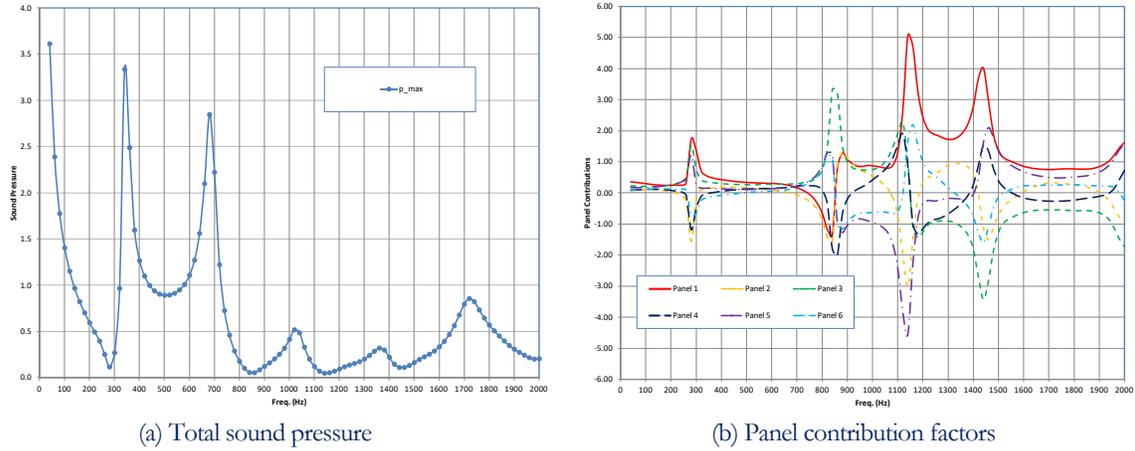


Figure 5.12. Sound pressure and panel contribution factors computed at the field point.

More verification and demonstration examples with more realistic and large-scale 3-D models, including video clips of the animations of the BEM results, can be found in Refs. [5, 12, 19], the flyer for the **FastBEM Acoustics**® package, and the *Verification Manual* at the website [www.fastbem.com](http://www.fastbem.com).

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